Mathematics Mastery

Progression in calculations

Year 1 – Year 6*

September 2017

NB. Users should familiarise themselves with the introduction (pp 2-10) to this document before referring to individual year group guidance.

**Progression guidance is not provided for EYFS/Reception since the focus should be on the understanding of early number concepts and number sense through the use of concrete manipulatives, as exemplified in the programmes of study.*

Introduction

At the centre of the mastery approach to the teaching of mathematics is the belief that all pupils have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, pupils must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This document outlines the different calculation strategies that should be taught and used in Years 1 to 6, in line with the requirements of the 2014 Primary National Curriculum.

Background

The 2014 Primary National Curriculum for mathematics differs from its predecessor in many ways. Alongside the end of Key Stage year expectations, there are suggested goals for each year; there is also an emphasis on depth before breadth and a greater expectation of what pupils should achieve.

One of the key differences is the level of detail included, indicating what pupils should be learning and when. This is suggested content for each year group, but schools have been given autonomy to introduce content earlier or later, with the expectation that by the end of each key stage the required content has been covered.

For example, in Year 2, it is suggested that pupils should be able to ‘add and subtract one-digit and two-digit numbers to 20, including zero’ and a few years later, in Year 5, they should be able to ‘add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction)’.

In many ways, these specific objectives make it easier for teachers to plan a coherent approach to the development of pupils’ calculation skills, and the expectation of using formal methods is rightly coupled with the explicit requirement for pupils to use multiple representations, including concrete manipulatives and images or diagrams – a key component of the mastery approach.

Purpose

The purpose of this document is threefold. Firstly, in this introduction, it outlines the structures for calculations, which enable teachers to systematically plan problem contexts for calculations to ensure pupils are exposed to both standard and non-standard problems. Secondly, it makes teachers aware of the strategies that pupils are formally taught within each year group, which will support them to perform mental and written calculations. Finally, it supports teachers in identifying appropriate pictorial representations and concrete materials to help develop understanding.

The policy only details the strategies; teachers must plan opportunities for pupils to apply these, for example, when solving problems, or where opportunities emerge elsewhere in the curriculum.

How to use the document

For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. Please note that the concrete and pictorial representation examples are not exhaustive, and teachers and pupils may well come up with alternatives. The purpose of using multiple representations is to give pupils a deep understanding of a mathematical concept and they should be able to work with and explain concrete, pictorial and abstract representations, and explain the links between different representations. Depth of understanding is achieved by moving between these representations. For example, if a child has started to use a pictorial representation, it does not mean that the concrete cannot be used alongside the pictorial. If a child is working in the abstract, depth can be evidenced by asking them to exemplify their abstract working using a concrete or pictorial representation and to explain what they have done using the correct mathematical vocabulary; language is, of course, one abstract representation but is given particular significance in the national curriculum.

Mathematical language

The 2014 National Curriculum is explicit in articulating the importance of pupils using the correct mathematical language as a central part of their learning. Indeed, in certain year groups, the non-statutory guidance highlights the requirement for pupils to extend their language around certain concepts.

> "The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof."
>
> *2014 Maths Programme of Study*

Suggested language structures accompany each strategy outlined in this document. These build on one another systematically, which supports pupils in making links between and across strategies as they progress through primary school. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, manipulatives, pictures or diagrams) and explained precisely. High expectations of the mathematical language used are essential, with teachers modelling accurate mathematical vocabulary and expecting pupils' responses to include it ***in full sentences.***

✓	✗
ones	units
is equal to	equals / makes
zero	oh (the letter O)

Presentation of calculations

You will see that throughout this document, calculations are presented in a variety of ways. It is important for pupils' mathematical understanding to experience and work with calculations and missing numbers in different positions relative to the = symbol. Examples used in classwork and independent work should reflect this.

Estimation

Pupils are expected to use their developing number sense from Year 1 to make predictions about the answers to their calculations. As their range of mental strategies increases, these predictions and, later, estimates should become increasingly sophisticated and accurate. All teaching of calculation should emphasise the importance of making and using these estimates to check, first, the sense and, later, the accuracy of their calculations.

Developing number sense

Fluency in arithmetic is underpinned by a good sense of number and an ability to understand numbers as both a concept (e.g. 7 is the value assigned to a set of seven objects) and as something resulting from a process (three beads and four more beads gives seven beads altogether or 3 + 4 = 7). Understanding that a number can be partitioned in many ways (e.g. 7 = 4 + 3; 5 + 2 = 7; 1 + 6 = 7) is key to being able to use numbers flexibly in calculating strategies. The part-whole model and, later, bar models, are particularly useful for developing a relational understanding of number. Pupils who are fluent in number bonds (initially within ten and then within twenty) will be able to use the 'Make ten' strategy efficiently, enabling them to move away from laborious and unreliable counting strategies, such as 'counting all' and 'counting on'. Increasing fluency in efficient strategies will allow pupils to develop flexible and interlinked approaches to addition and subtraction. At a later stage, applying multiplication and division facts, rather than relying on skip-counting, will continue to develop flexibility with number.

Structures and contexts for calculations

> *"In a technological age, in which most calculations are done on machines, it surely cannot be disputed that knowing which calculation to do is more important than being able to do the calculation."*
>
> Derek Haylock (2014); Mathematics Explained for Primary Teachers, p.56

There are multiple contexts (the word problem or real-life situation, within which a calculation is required) for each mathematical operation (i.e. addition) and, as well as becoming fluent with efficient calculating strategies, pupils also need to become fluent in identifying which operations are required. If they are not regularly exposed to a range of different contexts, pupils will find it difficult to apply their understanding of the four operations. For each operation, a range of contexts can be identified as belonging to one of the conceptual 'structures' defined below.

The **structure** is distinct from both the **operation** required in a given problem and the **strategy** that may be used to solve the calculation. In order to develop good number sense and flexibility when calculating, children need to understand that many strategies (preferably the most efficient one <u>for them</u>!) can be used to solve a calculation, once the correct operation has been identified. There is often an implied link between the given structure of a problem context and a specific calculating strategy. Consider the following question: A chocolate bar company is giving out free samples of their chocolate on the street. They began the day with 256 bars and have given away 197. How many do they have remaining? The reduction context implicitly suggests the action of 'taking away' and might lead to a pupil, for example, counting back or using a formal algorithm to subtract 197 from 256 (seeing the question as 256 – 197 = ☐). However, it is much easier to find the difference between 197 and 256 by adding on (seeing the question as 197 + ☐ = 256). Pupils with well-developed number sense and a clear understanding of the inverse relationship between addition and subtraction will be confident in manipulating numbers in this way.

Every effort is made to include multiple contexts for calculation in the Mathematics Mastery materials but, when teachers adapt the materials (which is absolutely encouraged), having an awareness of the different structures and being sure to include a range of appropriate contexts, will ensure that pupils continue to develop their understanding of each operation. The following list should not be considered to be exhaustive but defines the structures (and some suggested contexts) that are specifically included in the statutory objectives and the non-statutory guidance of the national curriculum. Specific structures and contexts are introduced in the Mathematics Mastery materials at the appropriate time, according to this guidance.

Importance of knowns vs unknowns and using part-whole understanding

One of the key strategies that pupils should use to identify the correct operation(s) to solve a given problem (in day-to-day life and in word problems) is to clarify the known and unknown quantities and identify the relationships between them. Owing to the inverse relationship between addition and subtraction, it is better to consider them together as 'additive reasoning', since changing which information is unknown can lead to either addition or subtraction being more suitable to calculate a solution for the same context. For the same reason, multiplication and division are referred to as 'multiplicative reasoning'. Traditionally, approaches involving key vocabulary have been the main strategy used to identify suitable operations but owing to the shared underlying structures, key words alone can be ambiguous and lead to misinterpretation (see for example the question below about Samir and Lena, where the key word 'less' might be identified, but addition is required to solve the problem).

A more effective strategy is to encourage pupils to establish what they know about the relationship between the known and unknown values and if they represent a part or the whole in the problem, supported through the use of part-whole models and/or bar models. In the structures exemplified below, the knowns and unknowns have been highlighted. Where appropriate, the part-whole relationships have also been identified. Pupils should always be given opportunities to identify and discuss these, both when calculating and when problem-solving.

Standard and non-standard contexts

Using key vocabulary as a means of interpreting problems is only useful in what are in this document defined as 'standard' contexts, i.e. those where the language is aligned with the operation used to solve the problem. Take the following example:

First there were 12 people on the bus. Then three ***more*** *people got on. How many people are on the bus now?*

Pupils would typically identify the word 'more' and assume from this that they need to add the values together, which in this case would be the correct action. However, in non-standard contexts, identifying key vocabulary is unhelpful in identifying a suitable operation. Consider this question:

First there were 12 people on the bus and then some more people got on at the school. Now there are 15 people on the bus. How many people got on at the school?

Again the word 'more' would be identified, and a pupil may then erroneously add together 12 and 15. It is therefore much more helpful to consider known and unknown values and the relations between them.

Overexposure to standard contexts and lack of exposure to non-standard contexts will mean pupils are more likely to rely on 'key vocabulary' strategies, as they see that this works in most of the cases they encounter. It is therefore important, when adapting lesson materials, that non-standards contexts are used systematically, alongside standard contexts.

Additive reasoning

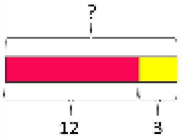
Change structures

augmentation (increasing)

where an existing value has been added to

Standard

First there were 12 people on the bus. Then three more people got on. How many people are on the bus now?

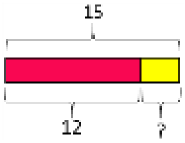


"I know both parts. My first part is twelve and my second part is three. I don't know the whole. I need to add the parts of twelve and three to find the whole."

12 + 3 = ?

Non-standard

First there were 12 people on the bus and then some more people got on at the school. Now there are 15 people on the bus. How many people got on at the school?

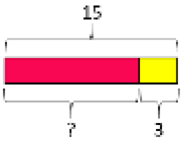


"I know my first part is twelve and I know the whole is 15. I don't know the value of the second part. To find the second part, I could add on from 12 to make 15 or I could subtract 12 from 15."

12 + ? = 15   15 – 12 = ?

Non-standard

First there were some people on the bus then it stopped to pick up three more passengers at the bank. Altogether now there are 15 people on the bus. How many were people were on the bus before it stopped at the bank?



"I know the value of the second part is three and that the whole is 15. I don't know the value of the first part. To find the first part, I could add on from three to make 15 or I could subtract three from 15."

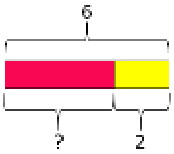
? + 3 = 15   15 – 3 = ?

reduction (decreasing)

where an existing value has been reduced

Standard

First Kieran had six plates in his cupboard. Then he took two plates out to use for dinner. How many plates are in the cupboard now?

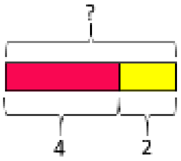


"I know the whole is six. I know one of the part that has been taken away is two. I don't know the other part. I need to subtract the known part, two, from the whole, six, to find the remaining part."

6 – 2 = ?   2 + ? = 6

Non-standard

First there were some plates in the cupboard. Then Kieran took two out for dinner. Now there are four left. How many plates were in the cupboard to start with?

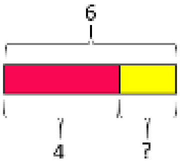


"I know the part that has been taken away is two and the part that is left is four. I don't know the whole. I can find the whole by adding the parts of four and two."

? – 2 = 4   2 + 4 = ?

Non-standard

First there were six plates in the cupboard. Then Kieran took some out for dinner. There are now four plates left in the cupboard. How many did Kieran take out?



"I know the whole is six and the remaining part is four. I don't know the part that was taken away. To find the part that was taken away I can add on from four to make six or I could subtract four from six."

6 – ? = 4   6 – 4 = ?

Note: the 'first... then... now' structure is used heavily in KS1 to scaffold pupils' understanding of change structures. Once pupils are confident with the structures, such linguistic scaffolding can be removed, and question construction can be changed to expose pupils to a greater range of nuance in interpreting problems. For example, the second and third reduction problems could be reworded as follows:

Kieran took two plates out of his cupboard for dinner. There were four left. How many plates were in the cupboard to begin with?

There were six plates in the cupboard before Kieran took some out for dinner. If there were four plates left in the cupboard, how many did Kieran take out?

These present the same knowns and unknowns, and therefore the same bar models and resulting equations to solve the problems; however, the change in wording makes them more challenging to pupils who have only worked with a 'first... then... now' structure so far.

Part-whole structures

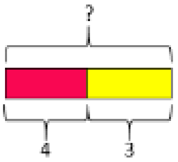
Combination (aggregation)/partitioning

combining two or more discrete quantities/splitting one quantity into two or more sub-quantities

Hakan and Sally have made a stack of their favourite books. Four books belong to Hakan, three to Sally. How many books are in the stack altogether?

"I know both parts. One part is four and the other part is three. I don't know the whole. I need to add the parts of three and four to find the whole."

4 + 3 = ?   3 + 4 = ?

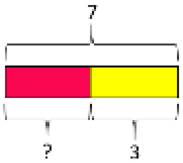


(Only one problem has been written for combination as, owing to the commutativity of addition, the only change in question wording would be to swap Hakan and Sally's names. The resulting bar model and calculation would be identical.)

Sally and Hakan have made a stack of their favourite books. There are seven books altogether. If three of them are Sally's, how many belong to Hakan?

"I know the whole is seven and that one of the parts is three. I don't know the other part. I need to add on from three to make seven or subtract three from seven to find the other part."

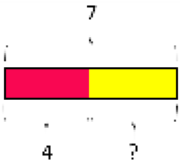
3 + ? = 7   7 – 3 = ?



Sally and Hakan have made a stack of their favourite books. There are seven books altogether. If four of them are Hakan's, how many belong to Sally?

"I know the whole is seven and that one of the parts is four. I don't know the other part. I need to add on from four to make seven or subtract four from seven to find the other part."

4 + ? = 7   7 – 4 = ?



Note: all part-whole contexts are considered to be 'standard', as the language of part-whole is unambiguous.

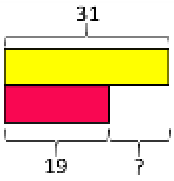
Comparison structures

Comparison structures involve a relationship between two quantities; their relationship is expressed as a difference. The structures vary by which of the values are known/unknown (the larger quantity, the smaller quantity and/or their difference). Part-whole language is not used here because the context contains not one single ‘whole’, but instead two separate quantities and it is the relationship between them being considered. Comparison bar models are therefore used to model these structures, which are known to be the most challenging for pupils to interpret.

Smaller quantity and larger quantity are known (comparative difference)

Standard

Navin has saved £19 from his pocket money. Sara has saved £31 from her pocket money. How much **more** has Sara saved than Navin? ***or*** How much **less** has Navin saved than Sara?



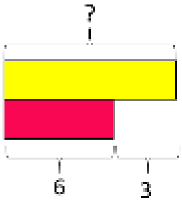
“I know one quantity is 19 and the other quantity is 31. I don’t know the difference. To find the difference I could add on from 19 to make 31 or I could subtract 19 from 31.”

19 + ? = 31   31 – 19 = ?

Smaller quantity and difference are known (comparative addition)

Standard

Ella has six marbles. Robin has three **more** than Ella. How many marbles does Robin have?

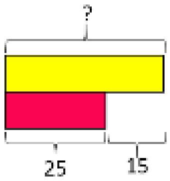


“I know the smaller quantity is six. I know the difference is three. I don’t know the larger quantity. To find the larger quantity I need to add three to six.”

6 + 3 = ?

Non-standard

Samir and Lena are baking shortbread but Lena’s recipe uses 15g **less** butter than Samir’s. If Lena needs to use 25g of butter, how much does Samir need?



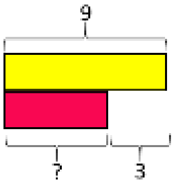
“I know the smaller quantity is 25. I know the difference between the quantities is 15. I don’t know the larger quantity. To find the larger quantity I need to add 15 to 25.”

? - 15 = 25   25 + 15 = ?

Larger quantity and difference are known (comparative subtraction)

Non-standard

Ella has some marbles. Robin has three **more** than Ella and he has nine marbles in total. How many marbles does Ella have?

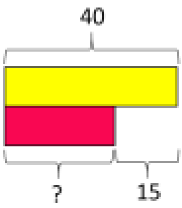


“I know the larger quantity is nine. I know the difference between the quantities is three. I don’t know the smaller quantity. To find the smaller quantity I need to add on from three to make nine or subtract three from nine.”

? + 3 = 9   9 – 3 = ?

Standard

Samir’s shortbread recipe uses 40g of butter. Lena’s recipe uses 15g **less** butter. How much butter does Lena need?



“I know one quantity is 40. I know the difference between the quantities is 15. I don’t know the smaller quantity but I know it is 15 less than 40. To find the smaller quantity, I need to subtract 15 from 40.”

40 – 15 = ?   ? + 15 = 40

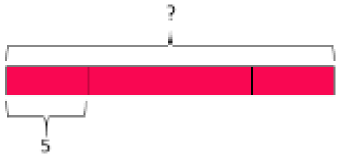
Multiplicative reasoning

Repeated grouping structures

repeated addition

groups (sets) of equal value are combined or repeatedly added

There are four packs of pencils. Each contains five pencils. How many pencils are there?



"I know there are four equal parts and that each part has a value of five. I don't know the value of the whole. To find the whole, I need to multiply four and five."

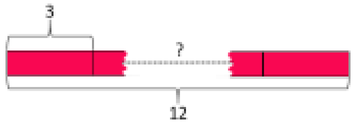
5 + 5 + 5 + 5 = ?

5 × 4 = ?

repeated subtraction (grouping)

groups (sets) of equal value are partitioned from the whole or repeatedly subtracted

There are 12 counters. If each child needs three counters to play the game, how many children can play?



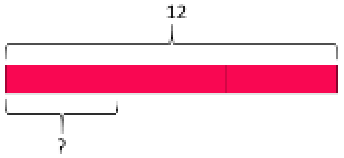
"I know the whole is twelve and that the value of each equal part is three. To find the number of equal parts, I need to know how many threes are in twelve."

3 × ? = 12   12 ÷ 3 = ?

sharing (into equal groups)

the whole is shared into a known number (must be a positive integer) of equal groups (sets)

Share twelve counters equally between three children. How many counters does each child get?



"I know the whole is twelve and the number of equal parts is three. I don't know the value of each part. To find the value of each part, I need to know what goes into twelve three times."

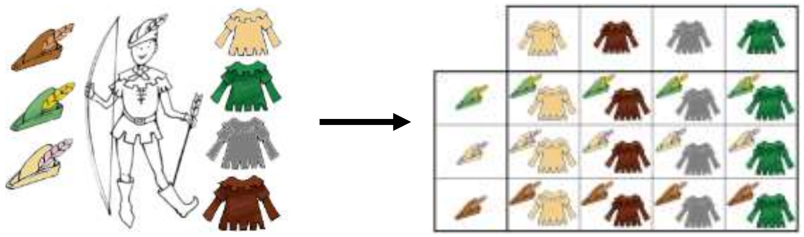
? × 3 = 12   12 ÷ 3 = ?

Cartesian product of two measures

correspondence

calculating the number of unique combinations that can be created from two (or more) sets

Robin has three different hats and four different tops. How many different outfits can he create, that combine one hat and one top?



"I know how many hats there are and I know how many tops there are. I don't know the number of different outfits that can be created. To find the number of outfits, I need to find how many different tops can be worn with each hat or how many different hats can be worn with each top."

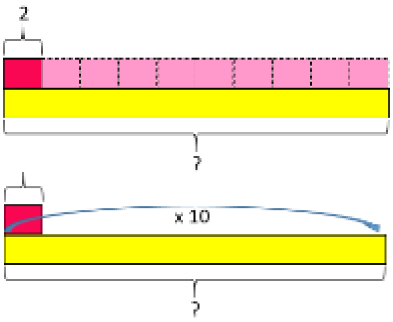
4 × 3 = ?   3 × 4 = ?

Scaling structures

scaling up (‘times greater/times as much’)

the original value is increased by a given scale factor

Rita receives £2 pocket money every week. Sim earns ten times as much money for her paper round. How much money does Sim earn?



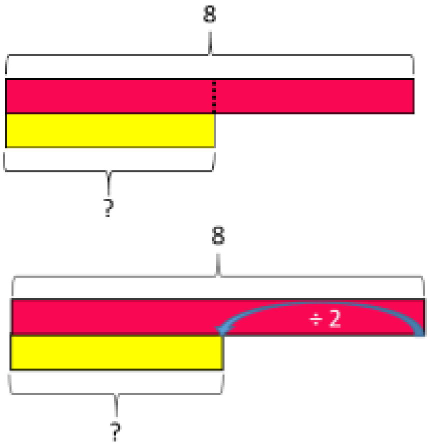
“I know one value is two and I know the second value is ten times greater. I don’t know the second value. To find the second value, I need to multiply two by ten.”

2 × 10 = ?

scaling down (‘times smaller/times less’)

the original value is reduced by a given scale factor

The house in my model village needs to be half the height of the church. If the church is 8 cm tall, how tall does the house need to be?



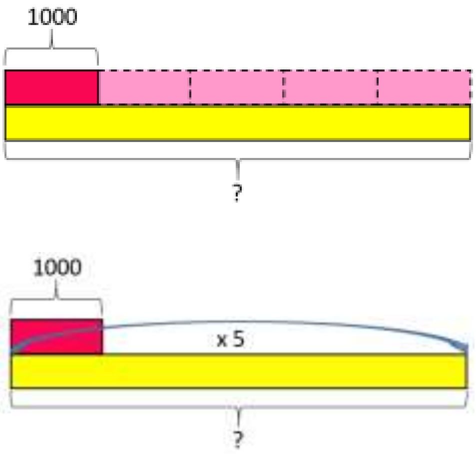
“I know one value is eight and I know the second value is half as great. I don’t know the second value. To find the second value, I need to halve eight (or divide it by two).”

Half of 8 is ? 8 ÷ 2 = ?

scaling up (‘times as many’)

the value of the original quantity is increased by a given scale factor

The Albert Hall can hold five times as many people as the Festival Hall. If the Festival Hall holds 1000 people, how many does the Albert Hall hold?



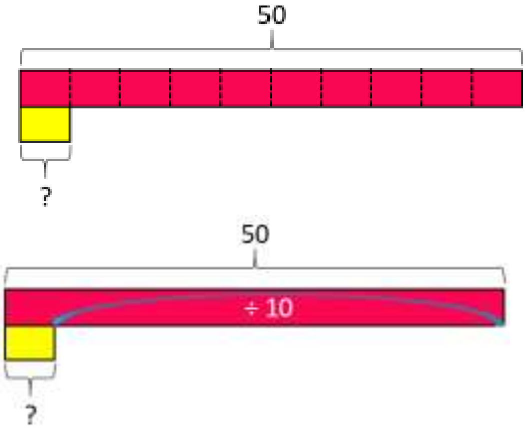
“I know one value is 1000 and I know the second value is five times greater. I don’t know the second value. To find the second value, I need to multiply 1000 by five.”

1000 × 5 = ?

scaling down (‘times fewer’)

the value of the original quantity is decreased by a given scale factor

Anouska’s garden pond has ten times fewer frogs than fish. If there are fifty fish, how many frogs are there?



“I know one value is 50 and I know the second value is ten times less. I don’t know the second value. To find the second value, I need to divide fifty by ten.”

50 ÷ 10 = ?

Progression in calculations

Year 1

National curriculum objectives linked to addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

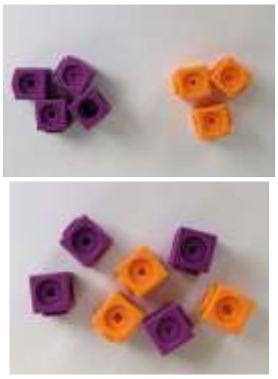
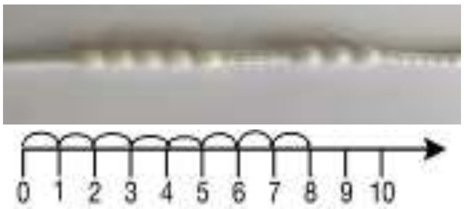
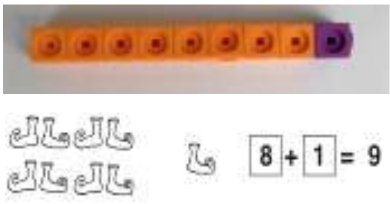
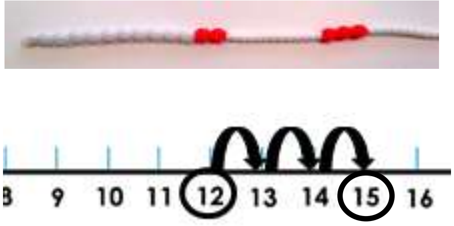
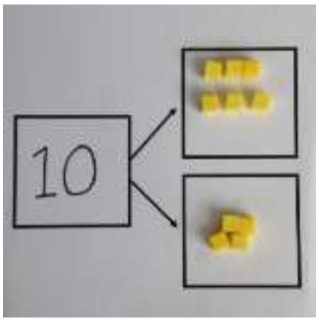
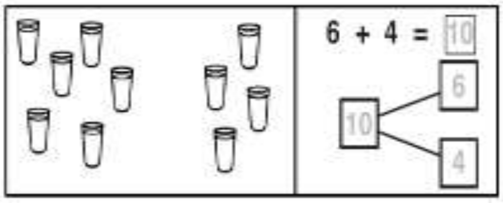
- Add and subtract one-digit and two-digit numbers to 100, including zero (N.B. Year 1 N.C. objective is to do this with numbers to 20).
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones, a two-digit number and tens, 2 two-digit numbers; add 3 one-digit numbers (Year 2).
- Represent and use number bonds and related subtraction facts within 20.
- Given a number, identify 1 more and 1 less.
- Show that addition of two numbers can be done in any order (commutative) but subtraction of one number from another cannot (Year 2).
- Recognise the inverse relationship between addition and subtraction and use this to solve missing number problems (Year 2).

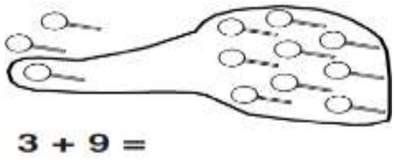
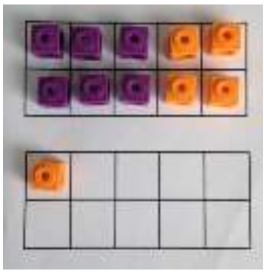
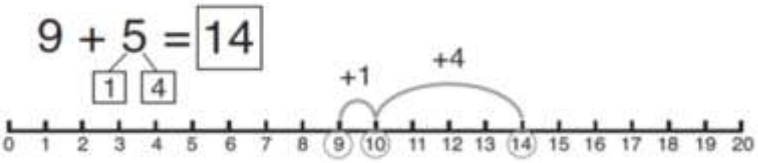
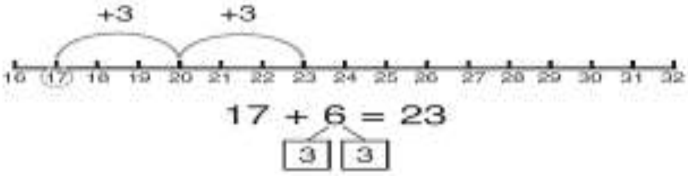
The following objectives should be planned for lessons where new strategies are being introduced and developed:

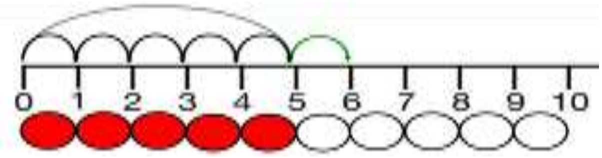
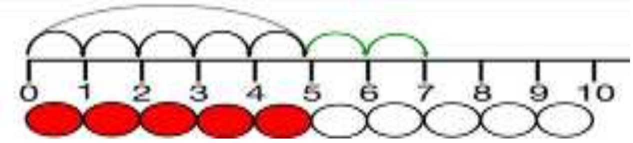
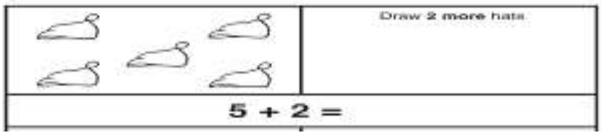
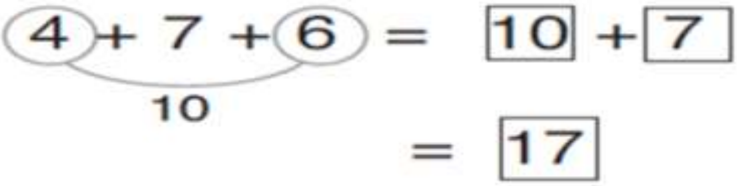
- Read, write and interpret mathematical statements involving addition (+), subtraction (−) and equal (=) signs.
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems, such as 7 = □ − 9.
- Solve problems with addition and subtraction:
  - Using concrete objects and pictorial representations, including those involving numbers, quantities and measures
  - Applying their increasing knowledge of mental methods

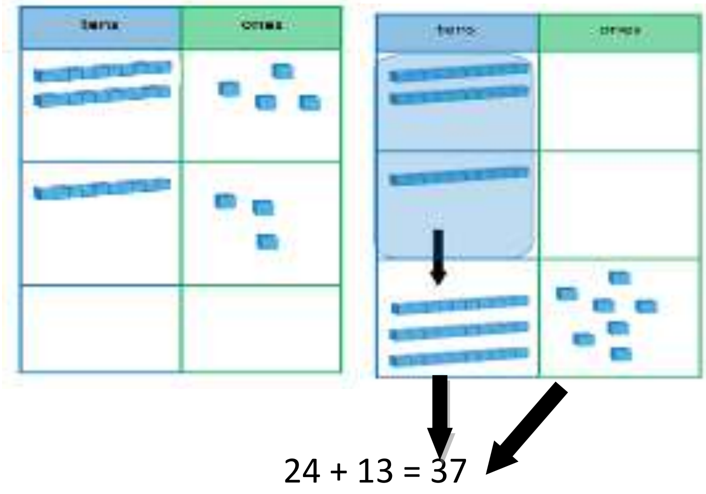
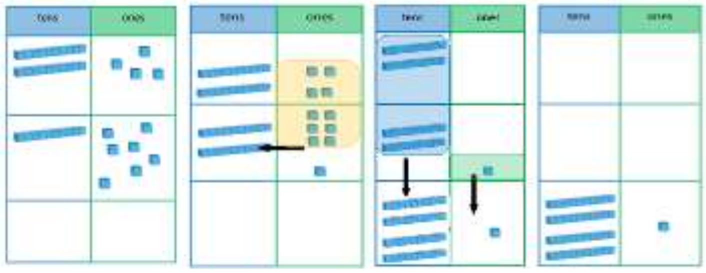
Teachers should refer to the definitions and guidance on the structures for addition and subtraction to provide a range of appropriate real-life contexts for calculations.

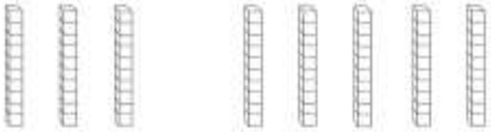
Y1 Addition

Strategy & guidance	CPA
Count all *Joining two groups and then recounting all objects using one-to-one correspondence*	$3 + 4 = 7$   $5 + 3 = 8$
Counting on *As a strategy, this should be limited to adding small quantities only (1, 2 or 3) with pupils understanding that counting on from the greater number is more efficient.*	$8 + 1 = 9$  $15 = 12 + 3$ 
Part-part-whole *Teach both addition and subtraction alongside each other, as pupils will use this model to identify the inverse relationship between them.* *This model begins to develop the understanding of the commutativity of addition, as pupils become aware that the parts will make the whole in any order.*	  $10 = 6 + 4$ $10 - 6 = 4$ $10 - 4 = 6$ $10 = 4 + 6$

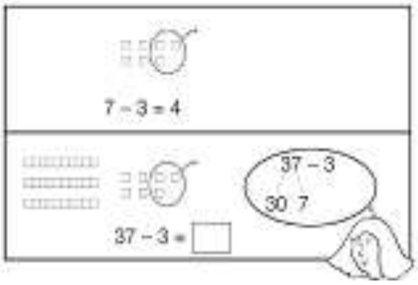
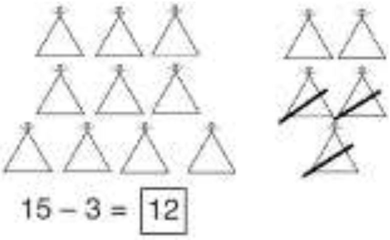
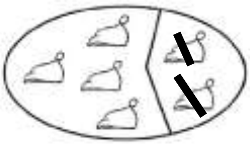
Regrouping ten ones to make ten *This is an essential skill that will support column addition later on.*	 3 + 9 = 12 
'Make ten' strategy *Pupils should be encouraged to start at the greater number and partition the smaller number to make ten.* *The colours of the beads on the bead string make it clear how many more need to be added to make ten.* *Also, the empty spaces on the ten frame make it clear how many more are needed to make ten.*	6 + 5 = 11  4 + 9 = 13   

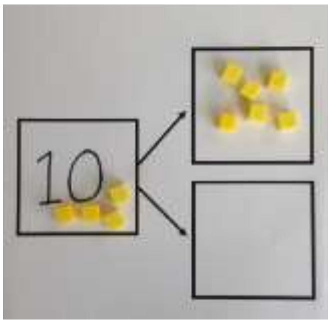
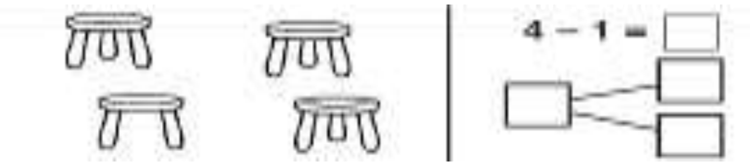
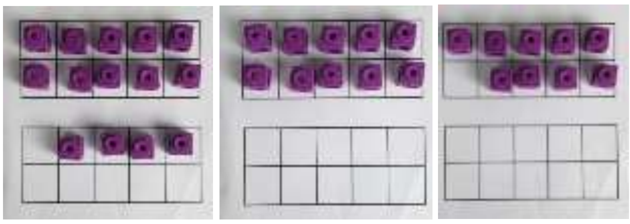
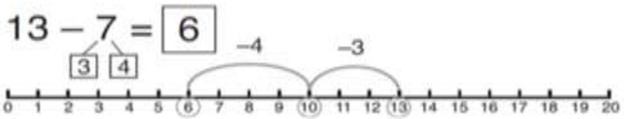
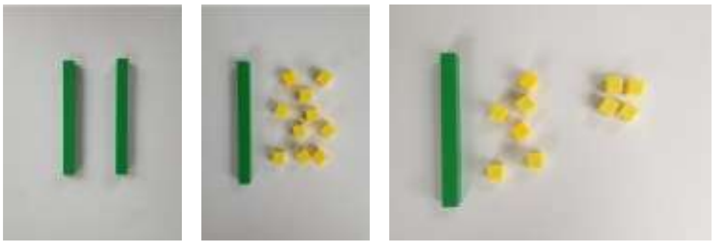
Adding 1, 2, 3 more *Here the emphasis should be on the language rather than the strategy. As pupils are using the beadstring, ensure that they are explaining using language such as;* *'1 more than 5 is equal to 6.'* *'2 more than 5 is equal to 7.'* *'8 is 3 more than 5.'* ***Over time, pupils should be encouraged to rely more on their number bonds knowledge than on counting strategies.***	1 more than 5 5 + 1 = 6  2 more than 5 5 + 2 = 7   
Adding three single digit numbers (make ten first) *Pupils may need to try different combinations before they find the two numbers that make 10.* *The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten.* *The second bead string shows 4, 6 and then 7.* *The final bead string shows how they have now been put together to find the total.*	 

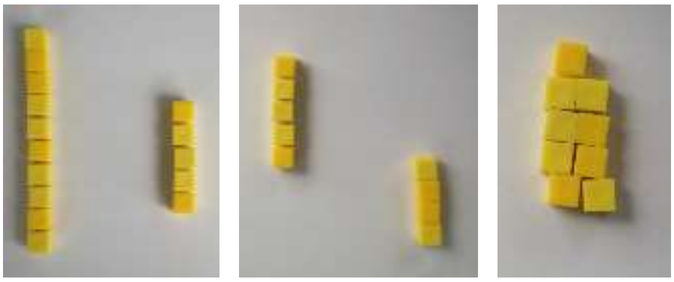
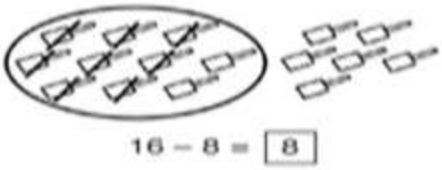
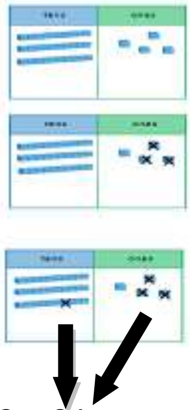
Partitioning to add (no regrouping) *Place value grids and Dienes blocks could be used as shown in the diagram before moving onto pictorial representations. Dienes blocks should always be available, as the main focus in Year 1 is the concept of place value rather than mastering the procedure.* *When not regrouping, partitioning is a mental strategy and does not need formal recording in columns. This representation prepares them for using column addition with formal recording.*	24 + 13 = 37 
Introducing column method for addition, regrouping only *Dienes blocks and place value grids should be used as shown in the diagrams. Even when working pictorially, pupils should have access to Dienes blocks.* *See additional guidance on unit pages for extra guidance on this strategy.*	24 + 17  Tens Ones 2 4 + 1 7 1 4 1 

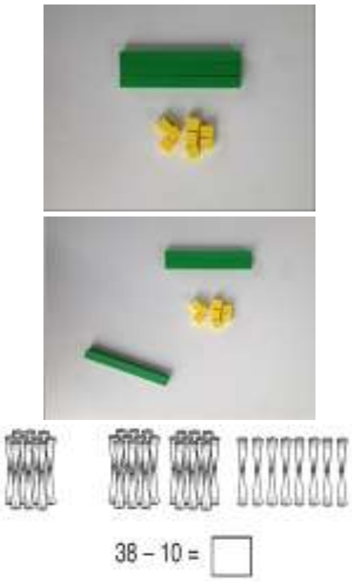
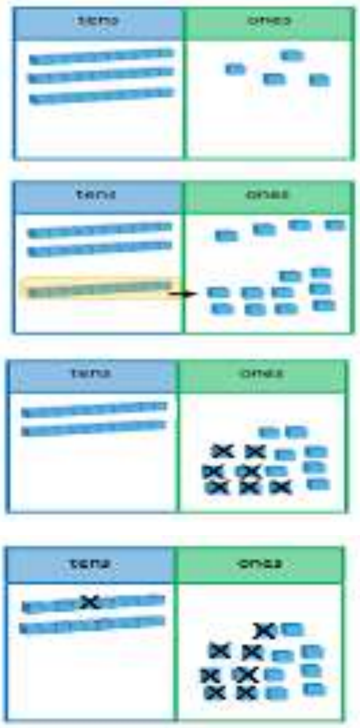
Adding multiples of ten *Using the vocabulary of 1 ten, 2 tens, 3 tens etc.* ***alongside*** *10, 20, 30 is important, as pupils need to understand that it is a* ***ten*** *and not a one that is being added and they need to understand that a '2' digit in the tens column has a value of twenty.* *It also emphasises the link to known number facts. E.g. '2 + 3 is equal to 5. So 2 tens + 3 tens is equal to 5 tens.*	50 = 30 + 20   3 tens + 5 tens = _____ tens 30 + 50 = _____  36 + 40 = $\boxed{}$

Y1 Subtraction

Strategy & guidance	CPA
Taking away from the ones *When this is first introduced, the concrete representation should be based upon the diagram. Real objects should be placed on top of the images as one-to-one correspondence so that pupils can take them away, progressing to representing the group of ten with a tens rod and ones with ones cubes.*	    28 - 4 =
Counting back *Subtracting 1, 2, or 3 by counting back* ***Pupils should be encouraged to rely on number bonds knowledge as time goes on, rather than using counting back as their main strategy.***	  4 = 6 – 2 16 – 2 = 14  

Part-part-whole *Teach both addition and subtraction alongside each other, as the pupils will use this model to identify the link between them. Pupils start with ten cubes placed on the whole.* *They then remove what is being taken away from the whole and place it on one of the parts.* *The remaining cubes are the other part and also the answer. These can be moved into the second part space.*	10 – 6 = 4  
Make ten strategy *To subtract a 1-digit number from a 2-digit number.* *Pupils identify how many need to be taken away to make ten first, partitioning the number being subtracted. Then they take away the rest to reach the answer.*	14 – 5 = 9  
Regroup a ten into 10 ones *After the initial introduction, the Dienes blocks should be placed on a place value chart to support place value understanding. This will support pupils when they later use the column method.*	  20 − 4 =

Taking away from the tens *Pupils should identify that they can also take away from the tens and get the same answer.* *This reinforces their knowledge of number bonds to 10 and develops their application of number bonds for mental strategies.*	9 = 15– 6  
Partitioning to subtract without regrouping *Dienes blocks on a place value chart (developing into using images on the chart) could be used, as when adding 2-digit numbers, reinforcing the main concept of place value for Year 1.* *When not regrouping, partitioning is a mental strategy and does not need formal recording in columns. This representation prepares them for using column subtraction with formal recording.*	34 – 13 = 21  34 – 13 = 21

Subtracting multiples of ten *Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important as pupils need to understand that it is a* ***ten*** *not a one that is being taken away.*	40 = 60 – 20  	38 – 10 = 28 
Column method with regrouping *This example shows how pupils should work practically when being introduced to this method.* *There is no formal recording in columns in Year 1 but this practical work will prepare pupils for formal methods in Year 2.* *See additional guidance on unit pages to support with this method.*	34 – 17 = 17  	

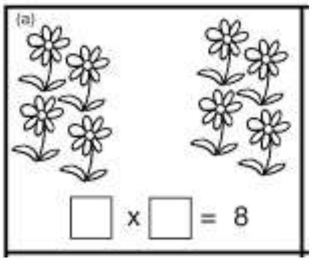
National Curriculum objectives linked to multiplication and division

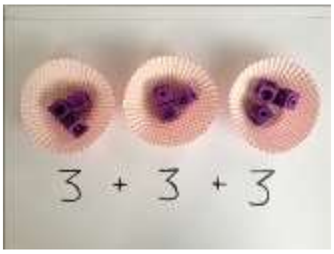
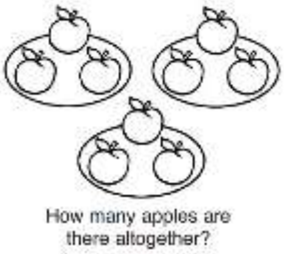
These objectives are explicitly covered through the strategies outlined in this document:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

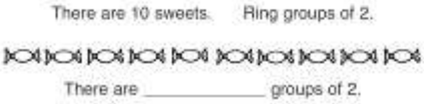
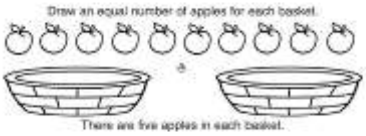
Teachers should refer to definitions and guidance on the structures for multiplication and division to provide a range of appropriate real-life contexts for calculations.

Y1 Multiplication

Strategy & guidance	**CPA**
Skip counting in multiples of 2, 5, 10 from zero *The representation for the amount of groups supports pupils' understanding of the written equation. So two groups of 2 are 2, 4. Or five groups of 2 are 2, 4, 6, 8, 10.* *Count the groups as pupils are skip counting.* *Number lines can be used in the same way as the bead string.* *Pupils can use their fingers as they are skip counting.*	 4 × 5 = 20  2 × 4 = 8
Making equal groups and counting the total *How this would be represented as an equation will vary. This could be 2 × 4 or 4 × 2. The importance should be placed on the vocabulary used alongside the equation. So this picture could represent 2 groups of 4 or 4 twice.*	  Draw  to show 2 x 3 = 6

Solve multiplications using repeated addition *This strategy helps pupils make a clear link between multiplication and division as well as exemplifying the 'repeated addition' structure for multiplication. It is a natural progression from the previous 'count all' strategy as pupils can be encouraged to 'count on'. However, as number bonds knowledge grows, pupils should rely more on these important facts to calculate efficiently.*	3 x 3 = 3 + 3 + 3   3 + 3 + 3 = 9

Y1 Division

Strategy & guidance	**CPA**
Sharing objects into groups *Pupils should become familiar with division equations through working practically.* *The division symbol is not formally taught at this stage.*	10 ÷ 2 = 5    

Progression in calculations

Year 2

National Curriculum objectives linked to addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

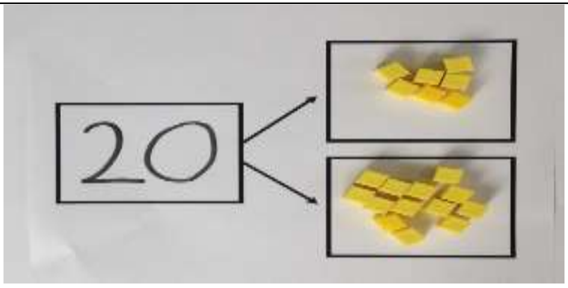
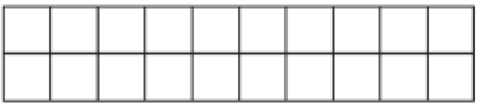
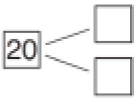
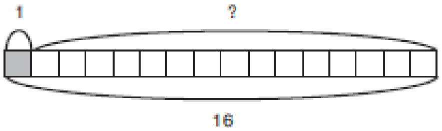
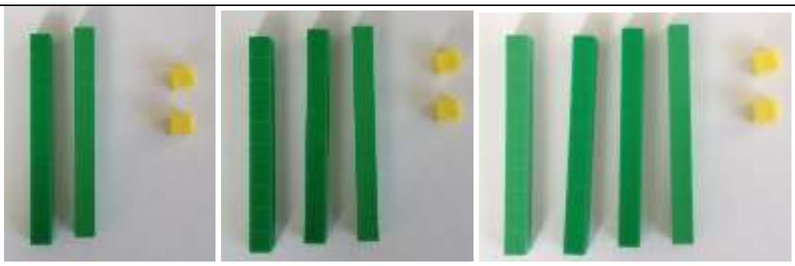
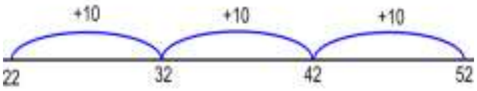
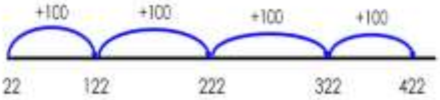
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; 2 two-digit numbers; adding three one-digit numbers.
- Add and subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three-digit number and hundreds (Year 3).
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Find 10 or 100 more or less than a given number (Year 3).
- Show that addition of two numbers can be done in any order (commutative) but subtraction of one number from another cannot.
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction (Year 3).

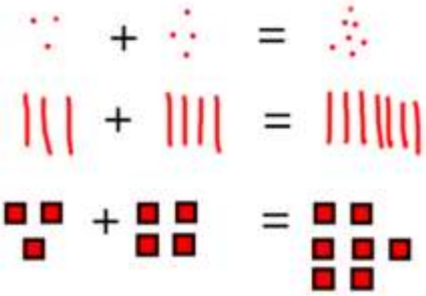
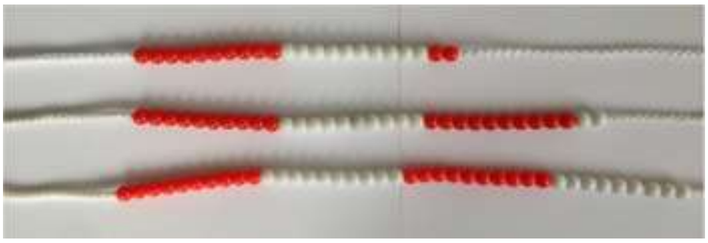
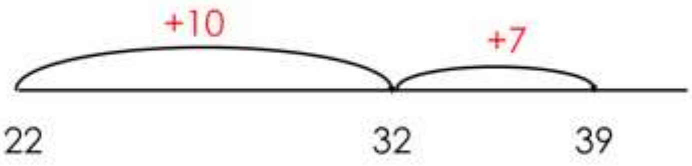
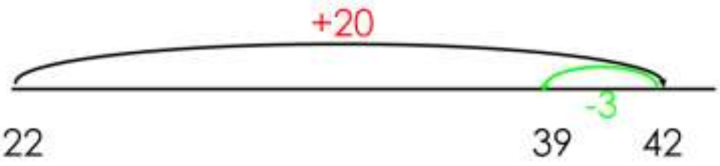
The following objectives should be planned for lessons where new strategies are being introduced and developed:

- Solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures; apply increasing knowledge of mental and written methods.
- Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction. (Year 3)

Teachers should refer to the definitions and guidance on the structures for addition and subtraction to provide a range of appropriate real-life contexts for calculations.

Y2 Addition

Strategy & guidance	**CPA**
Part-part-whole *Pupils explore the different ways of making 20. They can do this with all numbers using the same representations.* *This model develops knowledge of the inverse relationship between addition and subtraction and is used to find the answer to missing number problems.*	 20 = 17 + 3 20 = 3 + 17 20 – 3 = 17 20 – 17 = 3   ☐ + ☐ = 20 20 – ☐ = ☐ ☐ + ☐ = 20 20 – ☐ = ☐ ☐ + 1 = 16 16 – 1 = ☐ 1 + ☐ = 16 16 – ☐ = 1 
Counting on in tens and hundreds	  

Strategy & guidance	**CPA**
Using known facts to create derived facts *Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.*	 3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700
Partitioning one number, then adding tens and ones *Pupils can choose themselves which of the numbers they wish to partition. Pupils will begin to see when this method is more efficient than adding tens and taking away the extra ones, as shown.*	  22 + 17 = 39
Round and adjust (sometimes known as a compensating strategy) *Pupils will develop a sense of efficiency with this method, beginning to see when rounding and adjusting is more efficient than adding tens and then ones.*	  22 + 17 = 39

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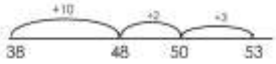
Make ten strategy



How pupils choose to apply this strategy is up to them; however, the focus should always be on efficiency.

It relies on an understanding that numbers can be partitioned in different ways in order to easily make a multiple of ten.

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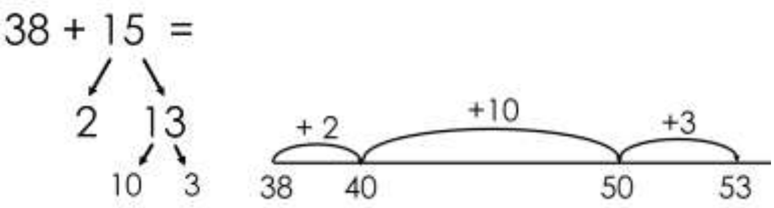

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Partitioning to add without regrouping

As in Year 1, this is a mental strategy rather than a formal written method. Pupils use the Dienes blocks (and later, images) to represent 3-digit numbers but do not record a formal written method if there is no regrouping.

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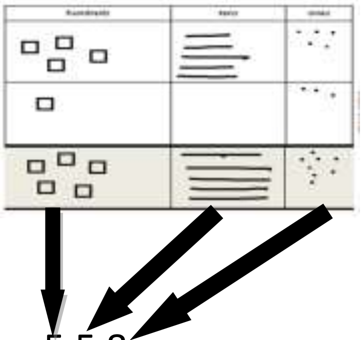

4 5 5 + 1 0 3 = 5 5 8

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Column method with regrouping

Dienes blocks should be used alongside the pictorial representations; they can be placed on the place value grid before pupils make pictorial representations.

As in Year 1, the focus for the column method is to develop a strong understanding of place value.

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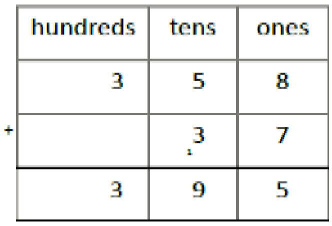
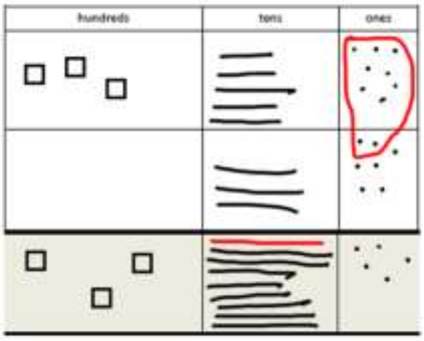
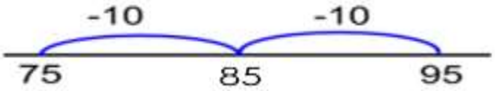
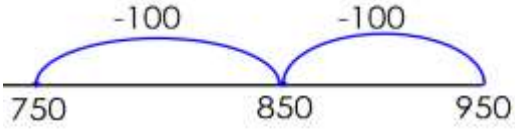
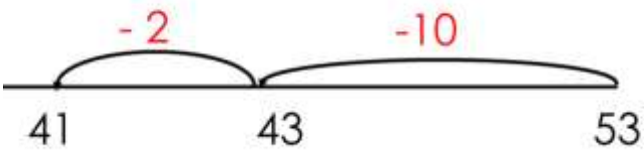


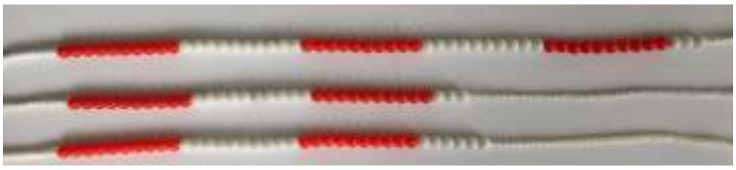
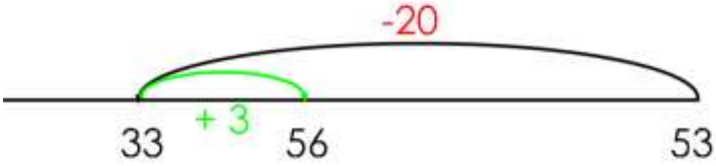
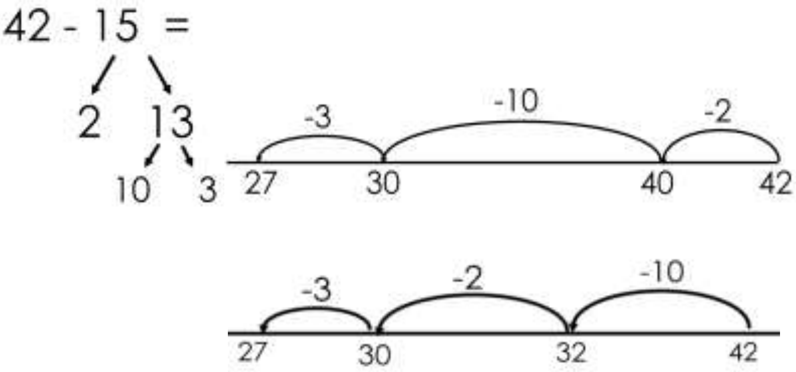
<table>
<tr><th></th><th>hundreds</th><th>tens</th><th>ones</th></tr>
<tr><td></td><td>3</td><td>5</td><td>8</td></tr>
<tr><td>+</td><td></td><td>3
1</td><td>7</td></tr>
<tr><td></td><td>3</td><td>9</td><td>5</td></tr>
</table>

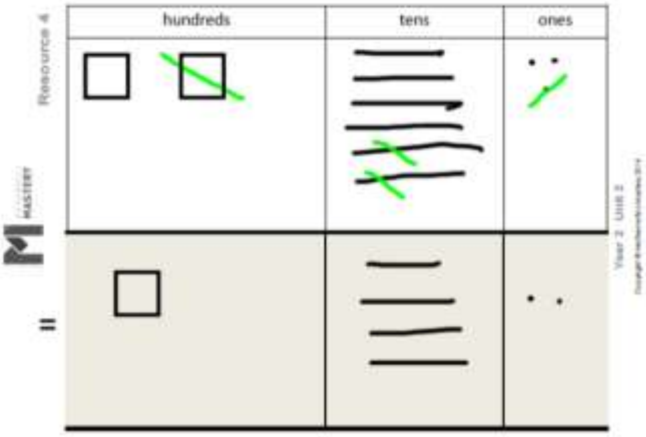
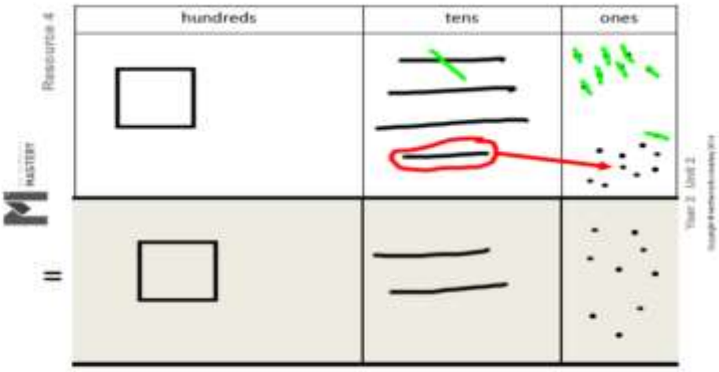


</td></tr>
</table>

Y2 Subtraction

Strategy & guidance	CPA
Counting back in multiples of ten and one hundred	  
Using known number facts to create derived facts *Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy, encouraging pupils to apply their knowledge of number bonds to add multiples of ten and 100.*	 8 – 4 = 4 *leads to* 80 – 40 = 40 *leads to* 800 – 400 = 400
Subtracting tens and ones *Pupils must be taught to partition the second number for this strategy as partitioning both numbers can lead to errors if regrouping is required.*	53 – 12 = 41  

Strategy & guidance	**CPA**
Round and adjust (sometimes known as a compensating strategy) *Pupils must be taught to round the number that is being subtracted.* *Pupils will develop a sense of efficiency with this method, beginning to identify when this method is more efficient than subtracting tens and then ones.*	  53 − 17 = 36
Make ten *How pupils choose to apply this strategy is up to them. The focus should always be on efficiency.* *It relies on an understanding that numbers can be partitioned in different ways in order to subtract to a multiple of ten.* *Pupils should develop an understanding that the parts can be added in any order.*	 

Strategy & guidance	CPA
Partitioning to subtract without regrouping *As in Year 1, the focus is to develop a strong understanding of place value and pupils should always be using concrete manipulatives alongside the pictorial.* *Formal recording in columns is unnecessary for this mental strategy. It prepares them to subtract with 3-digits when regrouping is required.*	 263 – 121= 142
Column method with regrouping *The focus for the column method is to develop a strong understanding of place value and concrete manipulatives should be used alongside.* *Pupils are introduced to calculations that require two instances of regrouping (initially from tens to one and then from hundreds to tens). E.g. 232 – 157 and are given plenty of practice using concrete manipulatives and images alongside their formal written methods, ensuring that important steps are not missed in the recording.* *Caution should be exercised when introducing calculations requiring 'regrouping to regroup' (e.g. 204 – 137) ensuring ample teacher modelling using concrete manipulatives and images.*	$\begin{array}{rrrr} & \text{hundreds} & \text{tens} & \text{ones} \\ & 1 & {}^{3}\cancel{4} & {}^{1}7 \\ - & & 1 & 8 \\ \hline & 1 & 2 & 9 \end{array}$ 

National Curriculum objectives linked to multiplication and division

These objectives are explicitly covered through the strategies outlined in this document:

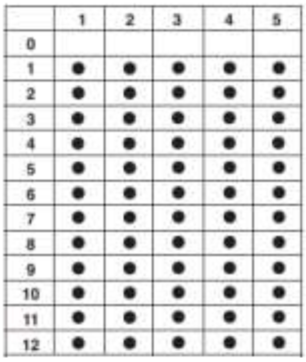
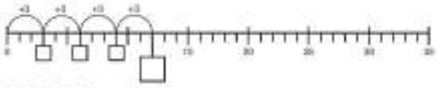
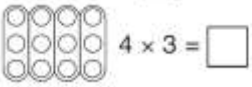
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Recall and use multiplication and division facts for the 3 and 4 multiplication tables (Year 3).
- Show that multiplication of two numbers can be done in any order (commutative) but division of one number by another cannot.

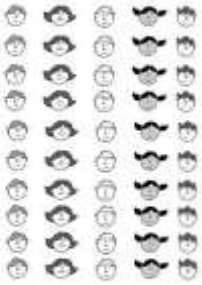
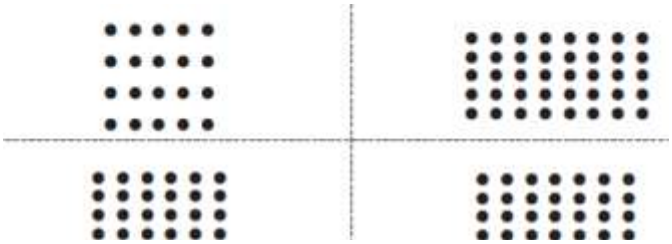
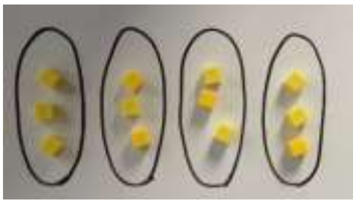
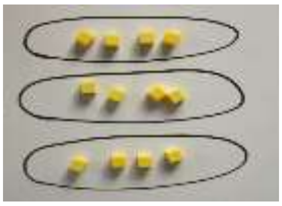
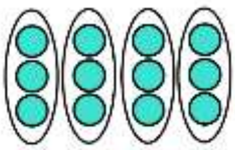
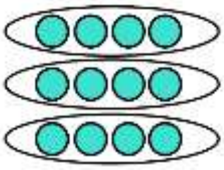
The following objectives should be planned for lessons where new strategies are being introduced and developed:

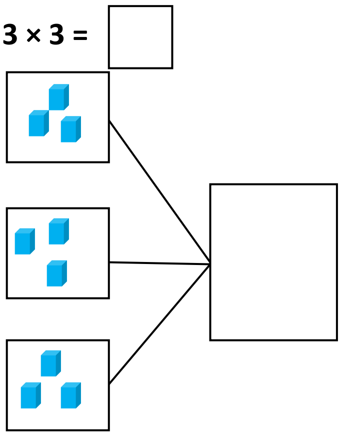
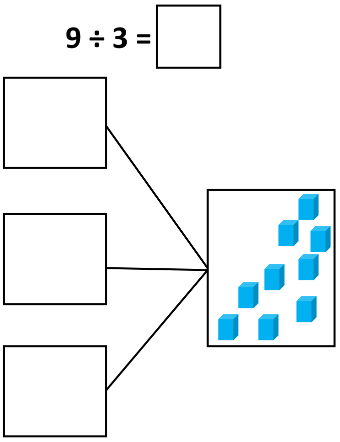
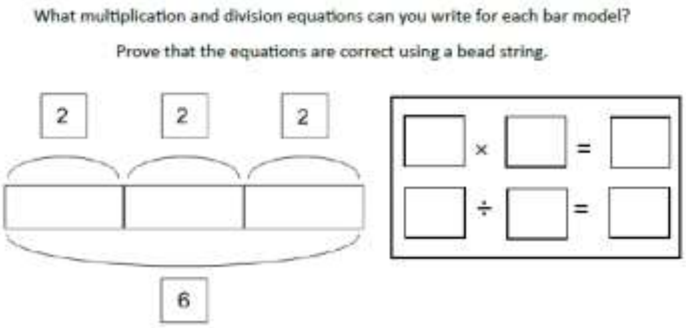
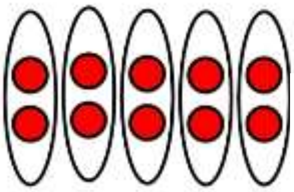
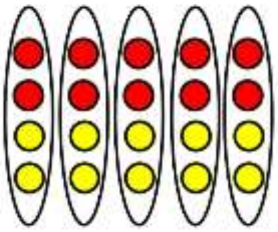
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equal (=) signs.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in context.

Teachers should refer to definitions and guidance on the structures for multiplication and division to provide a range of appropriate real-life contexts for calculations.

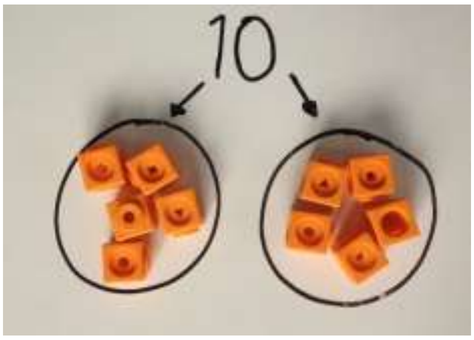
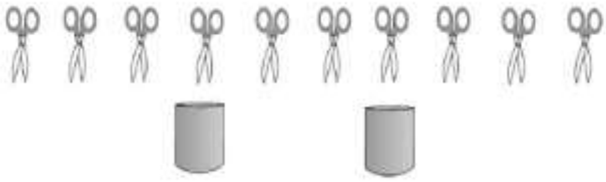
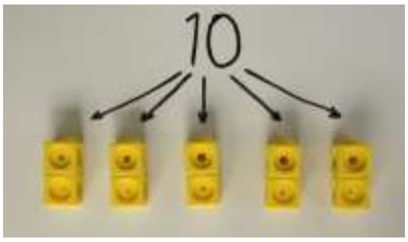
Y2 Multiplication

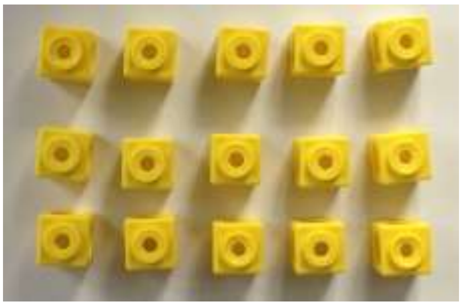
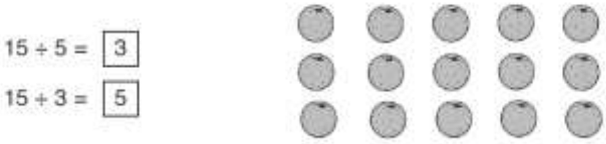
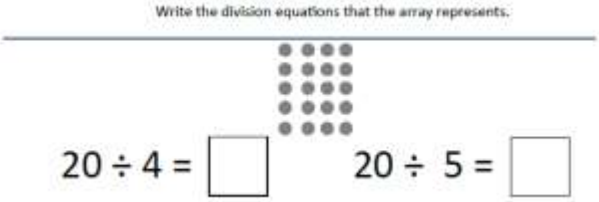
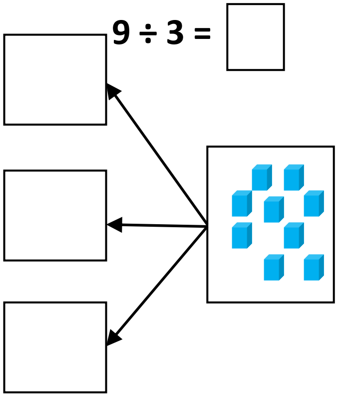
Strategy & guidance	CPA
Skip counting in multiples of 2, 3, 4, 5, 10 from zero *Pupils can use their fingers as they are skip counting, to develop an understanding of 'groups of'.* *Dot arrays can be used to create a visual representation for the different multiplication facts. Bead strings, groups of cubes (or unifix / multilink towers) provide useful concrete representations.*	 
Multiplication as repeated addition *Pupils apply skip counting to help find the totals of repeated additions.*	  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = ☐    4 × 3 = ☐

Strategy & guidance	CPA
Arrays to represent multiplication equations *Concrete manipulatives and images of familiar objects begin to be organised into arrays and, later, are shown alongside dot arrays. It is important to discuss with pupils how arrays can be useful.* *Pupils begin to understand multiplication in a more abstract fashion, applying their skip counting skills to identify the multiples of the 2x, 5x and 10x tables.* *The relationship between multiplication and division also begins to be demonstrated.*	   
Multiplication is commutative *Pupils should understand that an array and, later, bar models can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.*	 3 x 5 = ☐ 5 x 3 = ☐     12 = 3 × 4  12 = 4 × 3

Strategy & guidance	**CPA**
Use of part-part-whole model to establish the inverse relationship between multiplication and division *This link should be made explicit from early on, using the language of the part-part-whole model, so that pupils develop an early understanding of the relationship between multiplication and division. Bar models (with Cuisenaire rods) should be used to identify the whole, the value of the parts and the number of parts.* *It is important to highlight that with multiplication, the parts are of equal value as this is different to how this model is used for addition and subtraction.*	There are three equal parts. Each part has a value of three. What is the whole? **3 × 3 =**  **9 ÷ 3 =**   What multiplication and division equations can you write for each bar model? Prove that the equations are correct using a bead string.
Doubling to derive new multiplication facts *Pupils learn that known facts from easier multiplication tables can be used to derive facts from related times tables using doubling as a strategy.* *At this stage they double the 2× table facts to derive the 4× table facts.*	 5 × 2 = 10  5 × 4 = 20

Y2 Division

Strategy & guidance	**CPA**
Division as sharing *Here, division is shown as sharing.* *If we have ten pairs of scissors and we share them between two pots, there will be 5 pairs of scissors in each pot.*	10 ÷ 2 = 5  
Division as grouping *Here, division is shown as grouping.* *If we have ten forks and we put them into groups of two, there are 5 groups.*	10 ÷ 2 = 5  

Strategy & guidance	**CPA**
Use of part-part-whole model to represent division equations and to emphasise the relationship between division and multiplication *Pupils use arrays of concrete manipulatives and images of familiar objects to solve division equations.* *They begin to use dot arrays to develop a more abstract concept of division.* *It is important to highlight that with multiplication and division, the parts are of equal value as this is different to how this model is used for addition and subtraction.*	  15 ÷ 5 = 3 15 ÷ 3 = 5  Write the division equations that the array represents. 20 ÷ 4 = ☐ 20 ÷ 5 = ☐ The whole is nine. There are three equal parts. What is the value of each part?  **9 ÷ 3 =** ☐

Progression in calculations
Year 3

National Curriculum objectives linked to addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

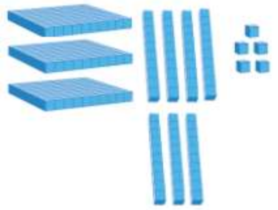
- add and subtract numbers mentally, including:
    - a three-digit number and ones
    - a three-digit number and tens
    - a three-digit number and hundreds
- add and subtract numbers with up to four digits, using formal written methods of columnar addition and subtraction (four digits is Year 4)
- find 10 or 100 more or less than a given number
- find 1 000 more or less than a given number (Year 4)
- estimate the answer to a calculation and use inverse operations to check answers

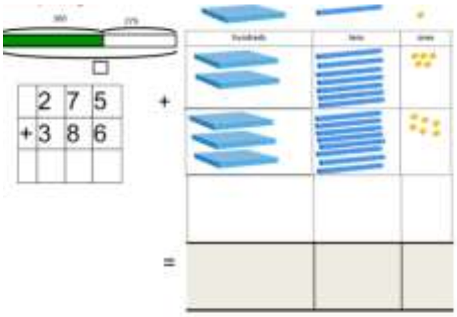
The following objectives should be planned for lessons where new strategies are being introduced and developed:

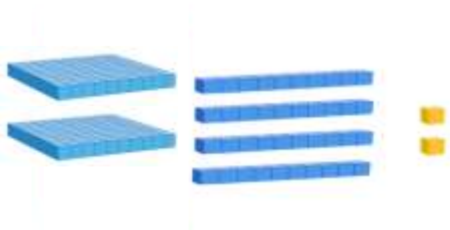
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

Teachers should refer to definitions and guidance on the structures for addition and subtraction to provide a range of appropriate real-life contexts for calculations.

Y3 Addition & Subtraction

Strategy & guidance	**CPA**
Add and subtract numbers mentally, including: - **a three-digit number and ones;** - **a three-digit number and tens;** - **a three-digit number and hundreds** *Pupils learn that this is an appropriate strategy when they are able to use known and derived number facts or other mental strategies to complete mental calculations with accuracy.* *To begin with, some pupils will prefer to use this strategy only when there is no need to regroup, using number facts within 10 and derivations. More confident pupils might choose from a range of mental strategies that avoid written algorithms, including (but not exhaustively):* - *known number facts within 20,* - *derived number facts,* - *'Make ten',* - *round and adjust* *See Year 2 guidance for exemplification of these – the use of concrete manipulatives other than Dienes blocks is important in reinforcing the use of these strategies.* *It is important that pupils are given plenty of (scaffolded) practice at choosing their own strategies to complete calculations efficiently and accurately. Explicit links need to be made between familiar number facts and the calculations that they can be useful for and pupils need to be encouraged to aim for efficiency.*	It is important to model the mental strategy using concrete manipulatives in the first instance and pupils should be able to exemplify their own strategies using manipulatives if required, with numbers appropriate to the unit they are working on (3-digit numbers in Units 1 & 4; 4-digit numbers in Unit 13). However, pupils should be encouraged to use known facts to derive answers, rather than relying on counting manipulatives or images. No regrouping 345 + 30 274 - 50 1128 + 300 1312 - 300 326 + 342 856 - 724  I know 4 + 3 = 7, so 4 tens plus 3 tens is equal to 7 tens. 345 + 30 = 375. With some regrouping 416 + 25 232 - 5 383 + 130 455 - 216 611 + 194 130 - 40 1482 + 900 2382 - 500

Strategy & guidance	CPA
Written column method for calculations that require regrouping with up to 4-digits *Dienes blocks should be used alongside the pictorial representations during direct teaching and can be used by pupils both for support and challenge. Place value counters can also be introduced at this stage.* *This work revises and reinforces ideas from Key Stage 1, including the focus on place value – see Year 2 exemplification.* *Direct teaching of the columnar method should require at least one element of regrouping, so that pupils are clear about when it is most useful to use it. Asking them 'Can you think of a more efficient method?' will challenge them to apply their number sense / number facts to use efficient mental methods where possible.* *As in Year 2, pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping. In Year 3 they become more familiar with calculations that require 'regrouping to regroup'. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.* *Pupils should be challenged as to whether this is the most efficient method, considering whether mental methods (such as counting on, using known number facts, round and adjust etc.) may be likelier to produce an accurate solution.* *Pupils requiring support might develop their confidence in the written method using numbers that require no regrouping.* *See Unit materials for extra guidance on this strategy.*	As for the mental strategies, pupils should be exposed to concrete manipulatives modelling the written calculations and should be able to represent their written work pictorially or with concrete manipulatives when required. Again, they should be encouraged to calculate with known and derived facts and should not rely on counting images or manipulatives.  5 + 6 = 11 so I will have 11 ones which I regroup for 1 ten and 1 one. Regrouping (including multiple separate instances) 672 + 136 734 – 82 468 + 67 831 - 76 275 + 386 435 – 188 'Regrouping to regroup' 204 – 137 1035 - 851

Strategy & guidance	CPA
Find 10, 100 more or less than a given number *As pupils become familiar with numbers up to 1000, place value should be emphasised and comparisons drawn between adding tens, hundreds (and, in the last unit of the Summer term, thousands), including use of concrete manipulatives and appropriate images.* *After initial teaching, this should be incorporated into transition activities and practised regularly.*	142 + 100 = 242  

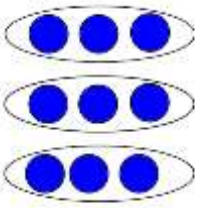
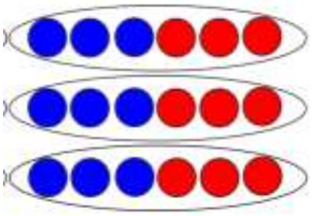
National Curriculum objectives linked to multiplication and division

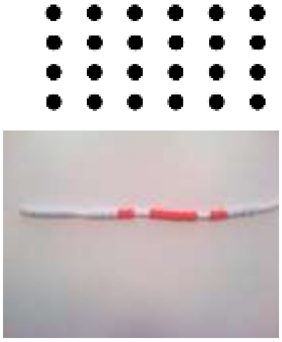
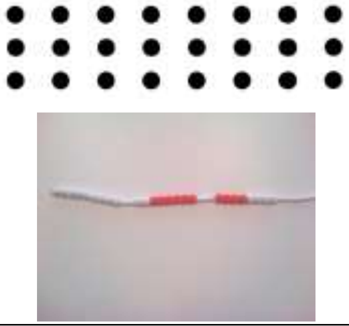
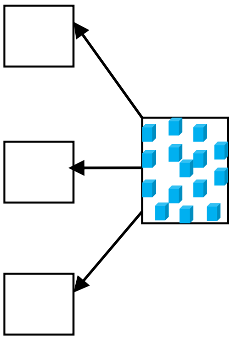
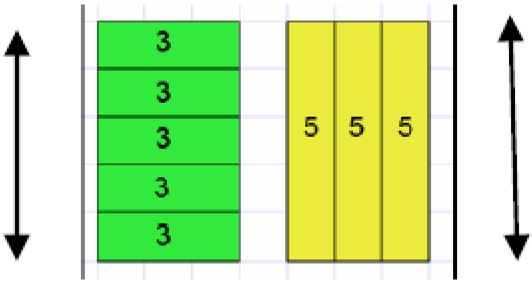
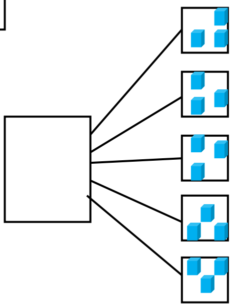
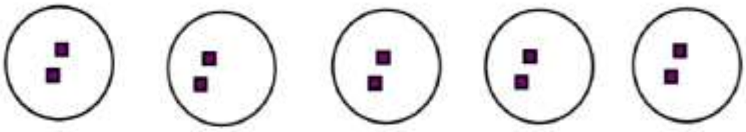
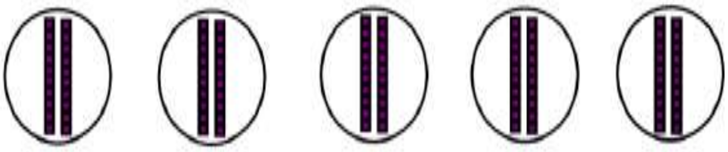
These objectives are explicitly covered through the strategies outlined in this document:

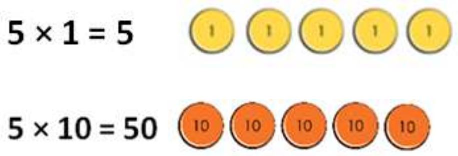
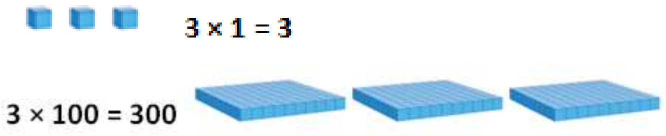
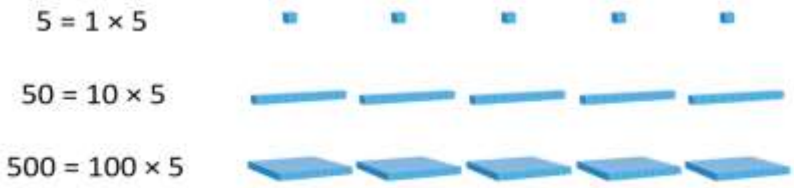
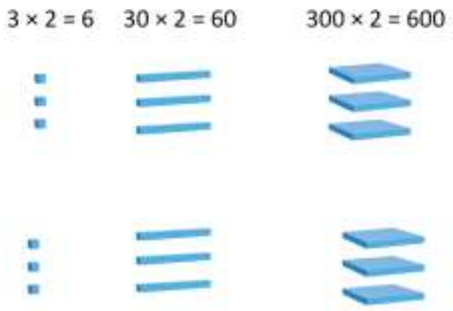
- count from 0 in multiples of 4, 8, 50 and 100
- recall and use multiplication and division facts for the 3, 4, 6, and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which *n* objects are connected to *m* objects

Teachers should refer to definitions and guidance on the structures for multiplication and division to provide a range of appropriate real-life contexts for calculations.

Y3 Multiplication

Strategy & guidance	**CPA**
Doubling to derive new multiplication facts *Pupils continue to make use of the idea that facts from easier times tables can be used to derive facts from related times tables using doubling as a strategy.* *This builds on the doubling strategy from Year 2.*	3 x 3 = 9  3 x 6 = double 9 = 18 

Strategy & guidance	CPA
Skip counting in multiples of 2, 3, 4, 5, 6, 8 and 10 *Rehearsal of previously learnt tables as well as new content for Year 3 should be incorporated into transition activities and practised regularly.*	 
Use of part-part-whole model with arrays and bar models to establish commutativity and inverse relationship between multiplication and division *In these contexts pupils are able to identify all the equations in a fact family.*	   
Ten times greater *Pupils's work on this must be firmly based on concrete representations – the language of ten times greater must be well modelled and understood to prevent the numerical misconception of 'adding a zero'.*	 

Strategy & guidance	CPA
Multiplying by 10 and 100 *Building on the ten times greater work, pupils use appropriate Dienes blocks and place value counters to multiply 2, 3, 4, 5 and 10 by 10, 100 and 1000.*	 
Using known facts for multiplying by multiples of 10 and 100 *Pupils' growing understanding of place value, allows them to make use of known facts to derive multiplications using powers of 10.* *It is important to use tables with which they are already familiar (i.e. not 7 or 9 tables in Year 3)*	 

<table>
<tr><th>Strategy & guidance</th><th>CPA</th></tr>
<tr><td>

Multiplication of 2-digit numbers with partitioning (no regrouping)

Children should always consider whether partitioning is the best strategy – if it is possible to use strategies such as doubling (some may use doubling twice for ×4), they need to choose the most efficient strategy.

Children may wish to make jottings, including a full grid as exemplified here – but grid method is not a formal method and its only purpose is to record mental calculations. This supports the development of the necessary mental calculating skills but does not hinder the introduction of formal written methods in Year 4. Concrete manipulatives are essential to develop understanding.

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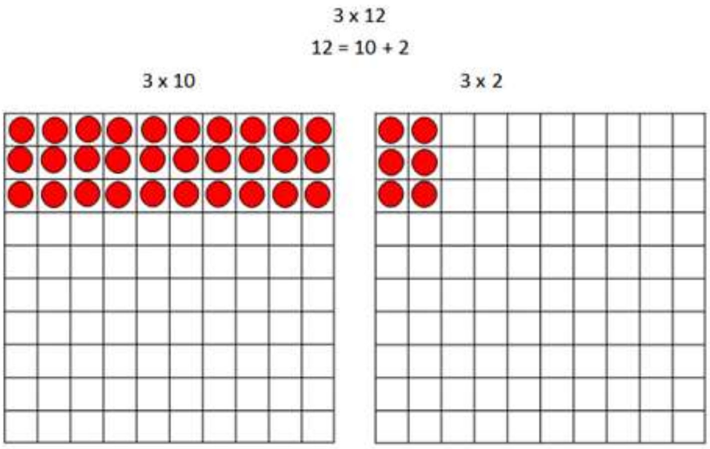
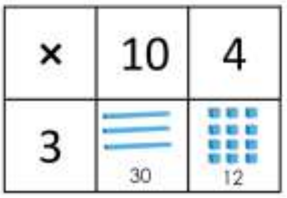
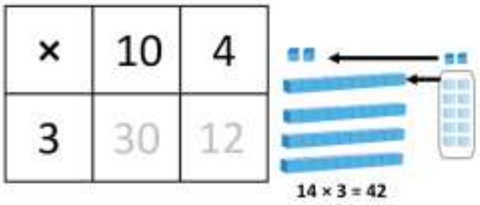
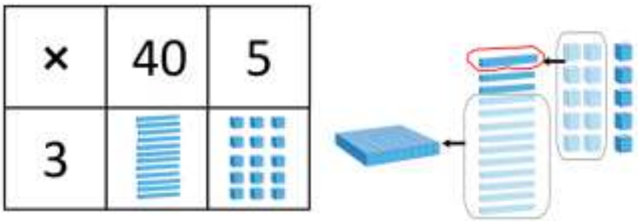


Now add the total number of tens and ones

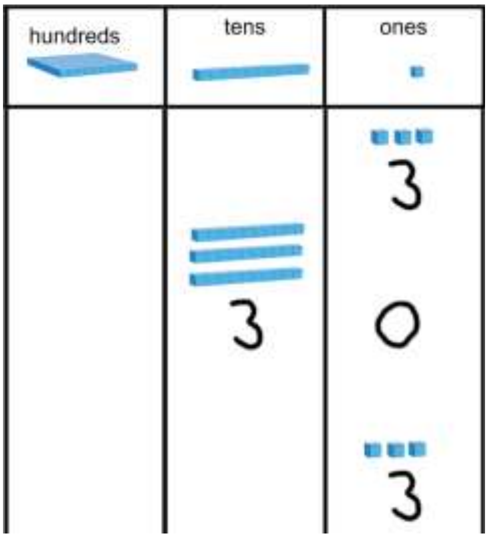
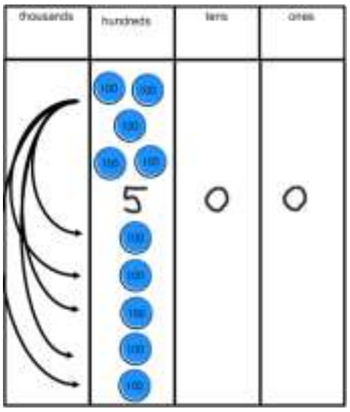
<table>
<tr><td>×</td><td>10</td><td>2</td></tr>
<tr><td>3</td><td>30</td><td>6</td></tr>
</table>

3 x 12 = 36

</td></tr>
</table>

Strategy & guidance	CPA
Multiplication of 2-digit numbers with partitioning (regrouping) *Using concrete manipulatives and later moving to using images that represent them, supports pupils' early understanding, leading towards formal written methods in Year 4.* *Once again, this is a mental strategy, which they may choose to support with informal jottings, including a full grid, as exemplified here.* *Pupils must be encouraged to make use of their known multiplication facts and their knowledge of place value to calculate, rather than counting manipulatives.*	  

Y3 Division

Strategy & Guidance	**CPA**
Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using scaling down *Pupils use the strategy of 'scaling down', representing numbers with concrete manipulatives and making the value ten times smaller.*	 **3 × 10 = 30** **30 ÷ 10 = 3**
Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using grouping *Pupils divide by 10, 100 and 1000 by making groups of the divisor.*	500 ÷ 100 = ☐ My whole is 500 and the value of the equal parts is 100. How many parts are there? 

Progression in calculations
Year 4

National curriculum objectives linked to addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

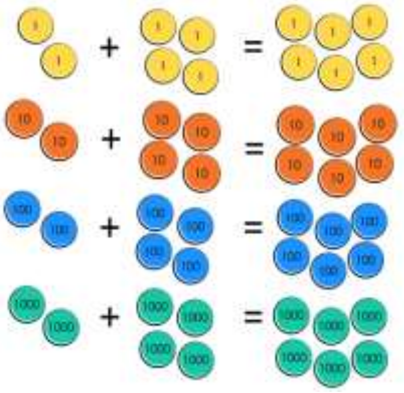
- add and subtract numbers with up to four digits, using the formal written methods of columnar addition and subtraction where appropriate
- find 1 000 more or less than a given number
- estimate and use inverse operations to check answers to a calculation

N.B. There is no explicit reference to mental calculation strategies in the programmes of study for Year 4 in the national curriculum. However, with an overall aim for fluency, appropriate mental strategies should always be considered before resorting to formal written procedures, with the emphasis on pupils making their own choices from an increasingly sophisticated range of strategies.

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why
- solve simple measure and money problems involving fractions and decimals to two decimal places

Y4 Addition & Subtraction

Strategies & Guidance	CPA
Count forwards and backwards in steps of 10, 100 and 1000 for any number up to 10 000. *Pupils should count on and back in steps of ten, one hundred and one thousand from different starting points. These should be practised regularly, ensuring that boundaries where more than one digit changes are included.* **Count forwards and backwards in tenths and hundredths**	  Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes. E.g. 990 + 10 or 19.9 + 0.1
Using known facts and knowledge of place value to derive facts. **Add and subtract multiples of 10, 100 and 1000 mentally** *Pupils extend this knowledge to mentally adding and subtracting multiples of 10, 100 and 1000. Counting in different multiples of 10, 100 and 1000 should be incorporated into transition activities and practised regularly.*	 2 + 4 = 6 20 + 40 = 60 200 + 400 = 600 2000 + 4000 = 6000
Adding and subtracting by partitioning one number and applying known facts. *By Year 4 pupils are confident in their place value knowledge and are calculating mentally both with calculations that do not require regrouping and with those that do.*	See Y3 guidance on mental addition & subtraction, remembering that use of concrete manipulatives and images in both teaching and reasoning activities will help to secure understanding and develop mastery.

<table>
<tr><th>Strategies & Guidance</th><th>CPA</th></tr>
<tr><td>

Round and adjust

Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding.

It is very easy to be confused about how to adjust and so visual representations and logical reasoning are essential to success with this strategy.

Build flexibility by completing the same calculation in a different order.

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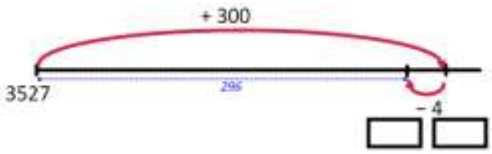
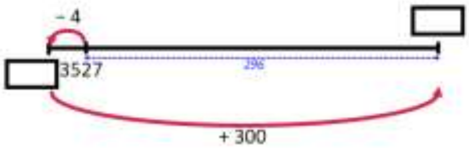
3527 + 296 = 3827 - 4

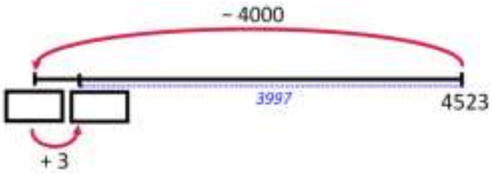


Completing the same calculation but adjusting first:

3527 + 296 = 3523 + 300

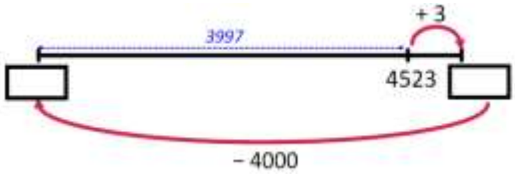


4523 – 3997 = 523 + 3



Completing the same calculation but adjusting first:

4523 – 3997 = 4526 - 4000



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<tr><td>

Near doubles

Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten. These facts can be adjusted to calculate near doubles.

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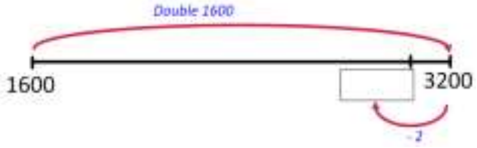
1600 + 1598 = double 1600 – 2



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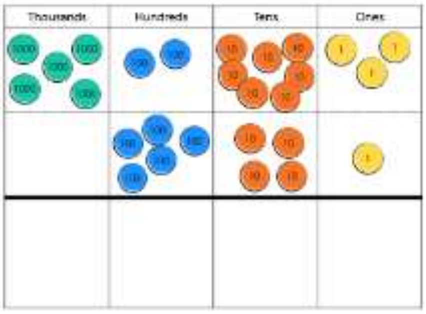
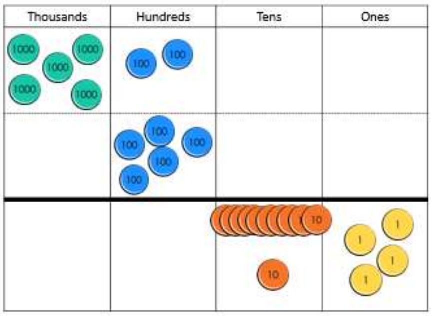
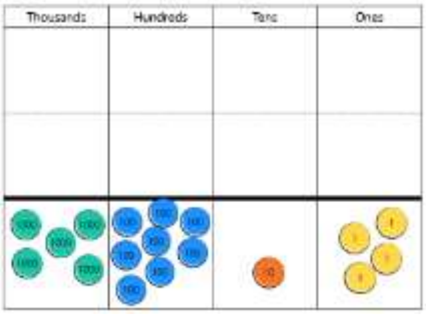
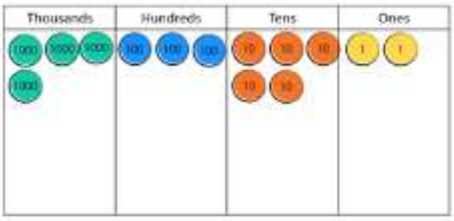
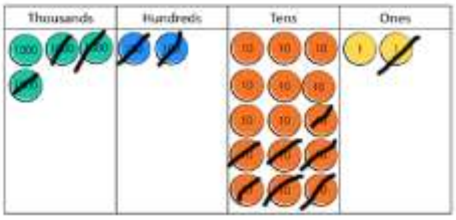
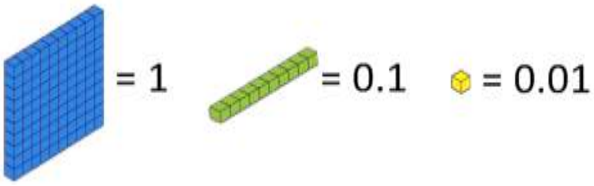
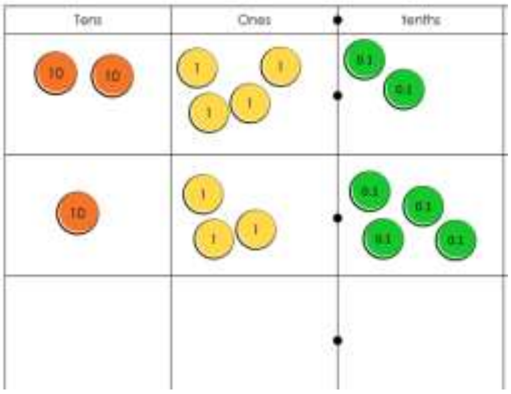
</table>

Strategies & Guidance	CPA
Written column methods for addition *Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.* *This method and the language to use are best understood through the tutorial videos found here on the toolkit.*	   $\begin{array}{ccccc} & 5 & 2 & 7 & 3 \\ + & & 5 & 4 & 1 \\ & & 1 & & \\ \hline & 5 & 8 & 1 & 4 \end{array}$
Written column methods for subtraction *Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.* *This method and the language to use are best understood through the tutorial videos on the toolkit.*	  $\begin{array}{ccccc} & 4 & \overset{2}{\cancel{3}} & {}^{1}5 & 2 \\ - & 3 & 2 & 7 & 1 \\ \hline & 1 & 0 & 8 & 1 \\ \hline \end{array}$

Strategies & Guidance	CPA
Calculating with decimal numbers *Assign different values to Dienes equipment. If a Dienes 100 block has the value of 1, then a tens rod has a value of 0.1 and a ones cube has a value of 0.01. These can then be used to build a conceptual understanding of the relationship between these.* *Place value counters are another useful manipulative for representing decimal numbers.* *All of the calculation strategies for integers (whole numbers) can be used to calculate with decimal numbers.*	 24.2 + 13.4 = 

National Curriculum objectives linked to multiplication and division

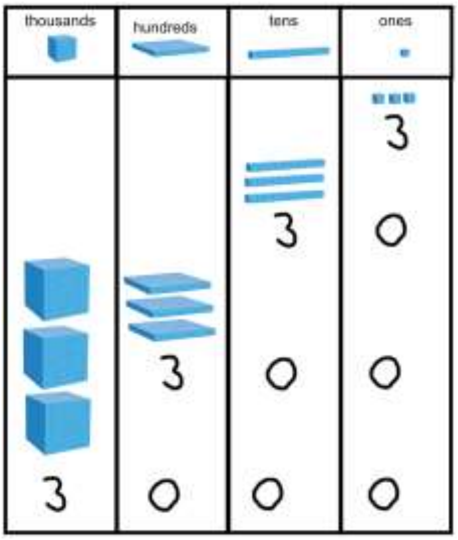
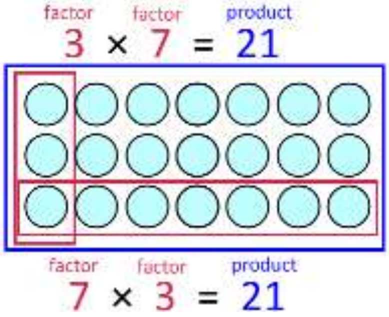
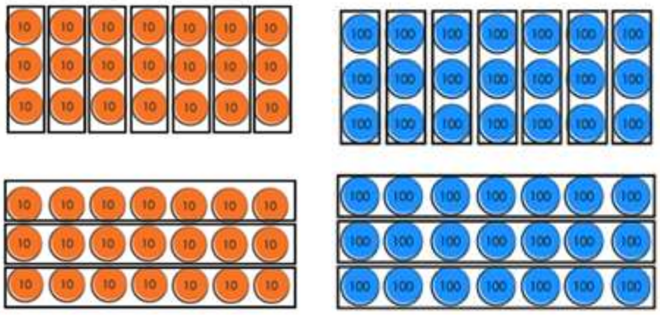
These objectives are explicitly covered through the strategies outlined in this document:

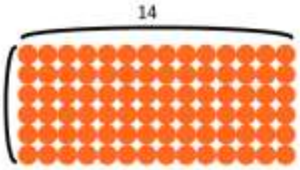
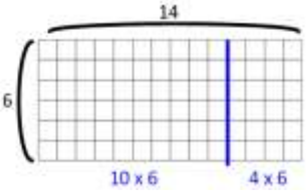
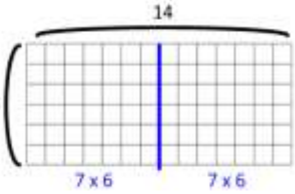
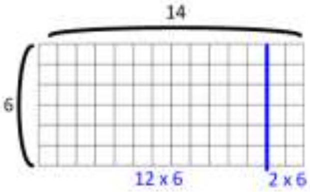
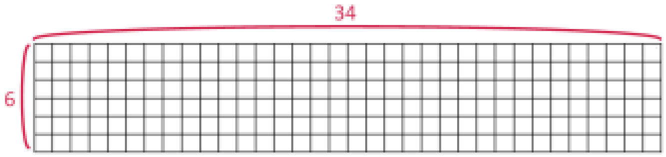
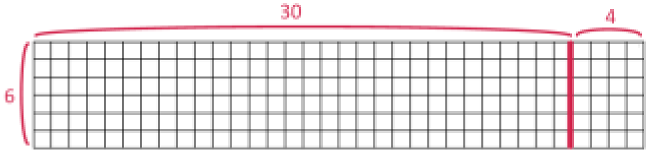
- count from 0 in multiples of 6, 7, 9, 25 and 1000
- recall and use multiplication and division facts for multiplication tables up to 12 × 12
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- recognise and use factor pairs and commutativity in mental calculations
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths.

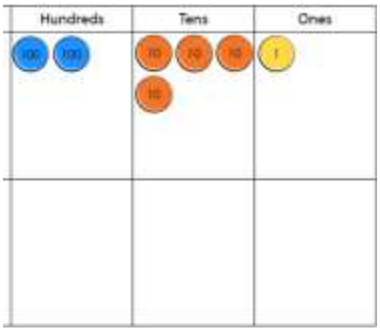
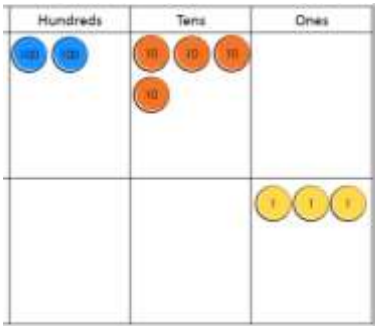
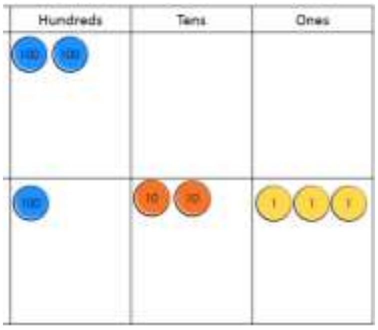
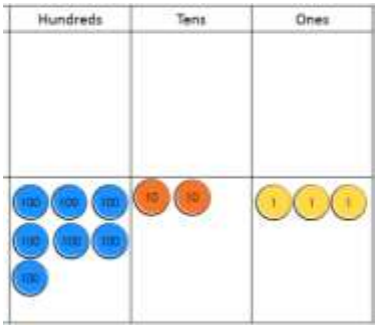
The following objectives should be planned for lessons where new strategies are being introduced and developed:

- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

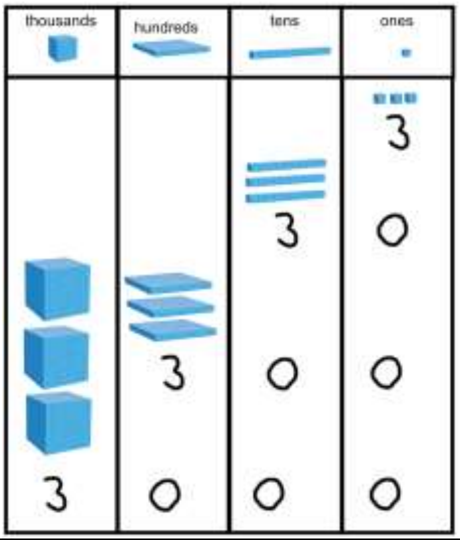
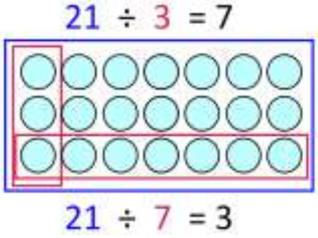
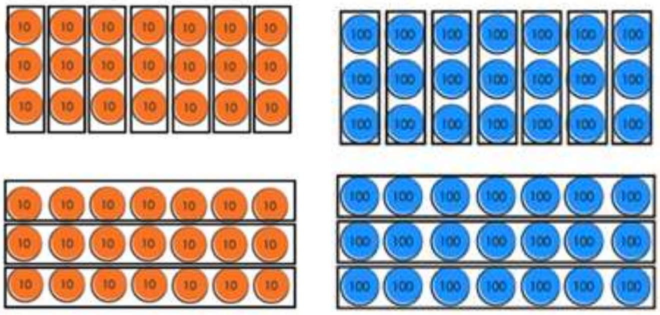
Y4 Multiplication

Strategies & Guidance	**CPA**
Multiplying by 10 and 100 *When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc.* *When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater.* *Repeated multiplication by ten will build an understanding of multiplying by 100 and 1000*	 3 x 10 = 30 3 x 100 = 300 3 x 1000 = 3000
Using known facts and place value for mental multiplication involving multiples of 10 and 100 *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact.* *Knowledge of commutativity (that multiplication can be completed in any order) is used to find a range of related facts.*	  30 x 7 = 210 300 x 7 = 2100 70 x 3 = 210 700 x 3 = 2100 7 x 30 = 210 7 x 300 = 2100 3 x 70 = 210 3 x 700 = 2100

Strategies & Guidance	**CPA**
Multiplying by partitioning one number and multiplying each part *Pupils build on mental multiplication strategies and develop an explicit understanding of distributive law, which allows them to explore new strategies to make more efficient calculations.* *As well as partitioning into tens and ones (a familiar strategy), they begin to explore compensating strategies and factorisation to find the most efficient solution to a calculation.* **Distributive law** **a x (b + c) = a x b + a x c**	14 x 6     34 × 6  30 × 6 + 4 × 6 
Mental multiplication of three 1-digit numbers, using the associative law *Pupils first learn that multiplication can be performed in any order, before applying this to choose the most efficient order to complete calculations, based on their increasingly sophisticated number facts and place value knowledge.*	Four pots each containing two flowers which each have seven petals. How many petals in total?  (4 x 2) x 7 or 4 x (2 x 7)

Strategies & Guidance	CPA
Short multiplication of 3-digit number by 1-digit number *To begin with pupils are presented with calculations that require no regrouping or only regrouping from the ones to the tens. Their conceptual understanding is supported by the use of place value counters, both during teacher demonstrations and during their own practice.* *With practice pupils will be able to regroup in any column, including from the hundreds to the thousands, including being able to multiply numbers containing zero and regrouping through multiple columns in a single calculation.* *This method and the language to use are best understood through the tutorial videos found here on the toolkit.*	Exemplification of this process is best understood through viewing the video tutorial  To calculate 241 x 3, represent the number 241. Multiply each part by 3, regrouping as needed.  $\begin{array}{cccc} & 2 & 4 & 1 \\ \times & & & 3 \\ \hline & 7 & 2 & 3 \\ \hline & 1 & & \end{array}$  

Y4 Division

Strategies & Guidance	**CPA**
Dividing by 10 and 100 *When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.* *When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. 210 ÷ 10 = 21*	 30 ÷ 10 = 3 300 ÷ 100 = 3 3000 ÷ 1000 = 3 300 ÷ 10 = 30 3000 ÷ 100 = 30 3000 ÷ 10 = 300
Derived facts *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.*	  210 ÷ 7 = 30 210 ÷ 3 = 70 210 ÷ 30 = 7 210 ÷ 70 = 3 2100 ÷ 7 = 300 2100 ÷ 3 = 700 2100 ÷ 300 = 7 2100 ÷ 700 = 3

<table>
<tr><th>Strategies & Guidance</th><th>CPA</th></tr>
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Short division of 4-digit numbers by 1-digit numbers

Pupils start with dividing 4-digit numbers by 2, 3 and 4, where no regrouping is required. Place value counters are used simultaneously in a place value chart, to develop conceptual understanding.

They progress to calculations that require regrouping in the hundreds or tens columns.

Pupils build on their conceptual knowledge of division to become confident with dividing numbers where the tens digit is smaller than the divisor, extending this to any digit being smaller than the divisor.

Exemplification of this method and the language to use are best understood through viewing the tutorial videos found here on the toolkit.

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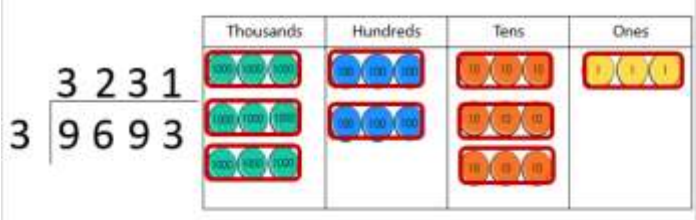
Division as sharing

Division as grouping



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<tr><td>

Division of a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths

When you divide by ten, each part is ten times smaller. The tens become ones and the ones become tenths. Each digit is in a place that gives it a value that is ten times smaller.

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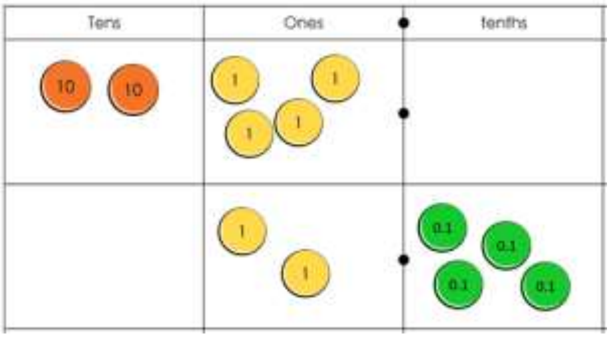
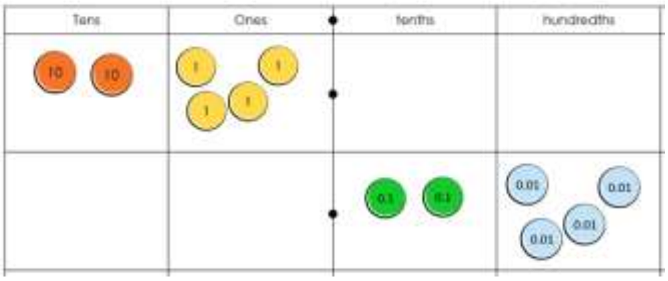
24 ÷ 10 = 2.4



24 ÷ 100 = 0.24



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Mathematics Mastery

Progression in calculations

Year 5 + Year 6

Year 5 and Year 6 are together because the calculation strategies used are broadly similar, with Year 6 using larger and smaller numbers. Any differences for Year 6 are highlighted in red.

National Curriculum objectives linked to integer addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

- add and subtract numbers mentally with increasingly large numbers
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- use negative numbers in context, and calculate intervals across zero
- perform mental calculations, including with mixed operations and large numbers
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.

Y5 and Y6 Addition & Subtraction

<table>
<tr><th>Strategies & Guidance</th><th>CPA</th></tr>
<tr><td>

Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000

Skip counting forwards and backwards in steps of powers of 10 (i.e. 10, 100, 1000, 10 000 and 100 000) should be incorporated into transition activities and practised regularly.

In Year 5 pupils work with numbers up to 1 000 000 as well as tenths, hundredths and thousandths.

In Year 6 pupils work with numbers up to 10 000 000.

</td><td>

Support with place value counters on a place value chart, repeatedly adding the same counter and regrouping as needed.

<table>
<tr><th>Hundred Thousands</th><th>Ten Thousands</th><th>Thousands</th><th>Hundreds</th><th>Tens</th><th>Ones</th><th>tenths</th><th>hundredths</th><th>thousandths</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Counting sticks and number lines:

Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes.

e.g. 9900 + 100 = 10 000 or 99 000 + 1000 = 100 000

</td></tr>
<tr><td>

Using known facts and understanding of place value to derive

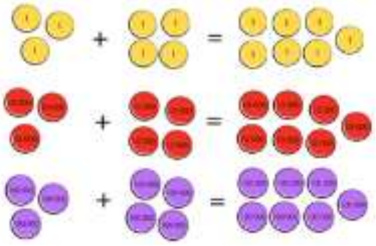
Using the following language makes the logic explicit: I know three ones plus four ones is equal to seven ones. Therefore, three ten thousands plus four ten thousands is equal to seven ten thousands.

In Year 5 extend to multiples of 10 000 and 100 000 as well as tenths, hundredths and thousandths.

In Year 6 extend to multiples of one million.

These derived facts should be used to estimate and check answers to calculations.

</td><td>

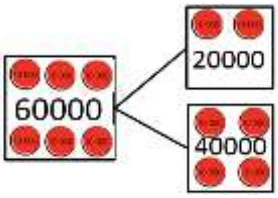


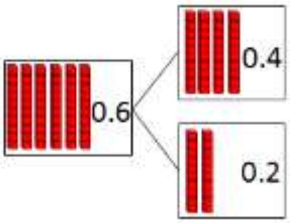
3 + 4 = 7

30 000 + 40 000 = 70 000

300 000 + 400 000 = 700 000

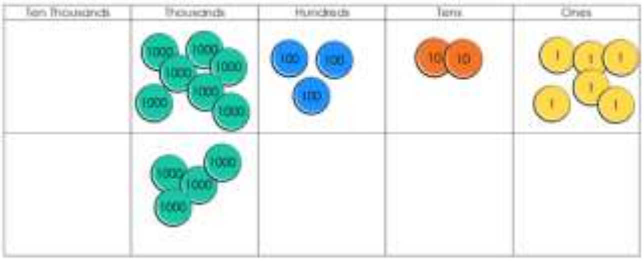
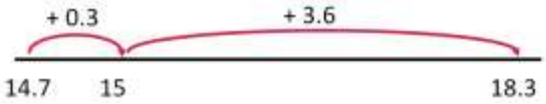
20 000 + 40 000 = 60 000
40 000 + 20 000 = 60 000
60 000 - 40 000 = 20 000
60 000 - 20 000 = 40 000

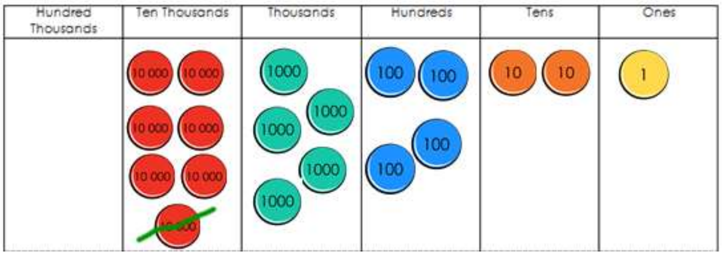
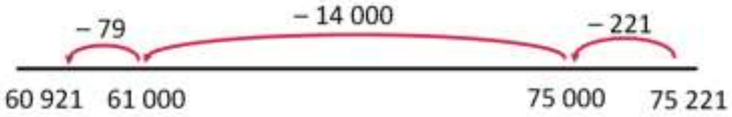


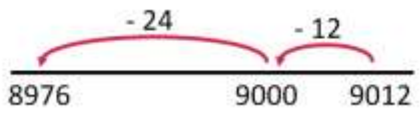
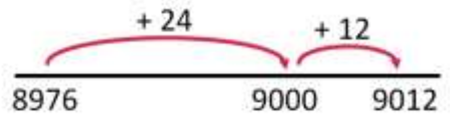


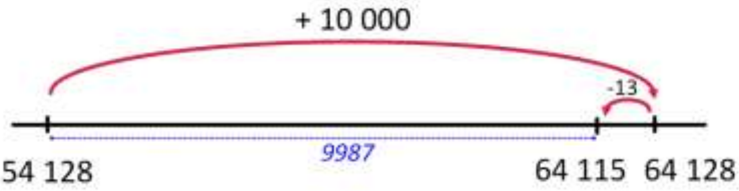
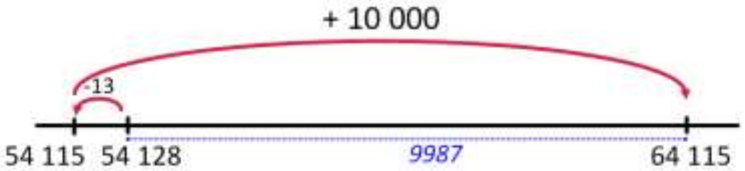
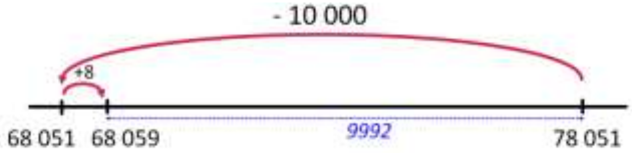
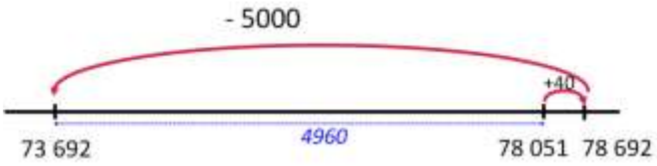
0.6 = 0.2 + 0.4
0.6 = 0.4 + 0.2
0.2 = 0.6 - 0.4
0.4 = 0.6 - 0.2

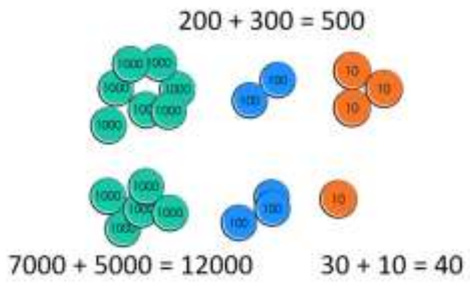
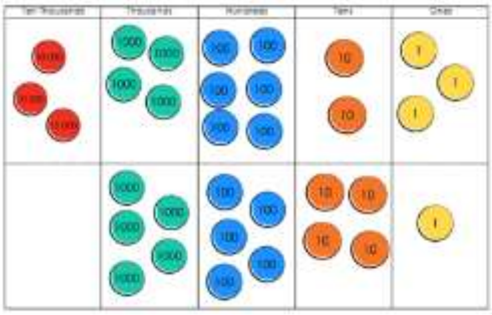
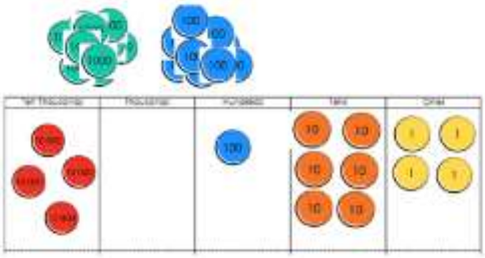
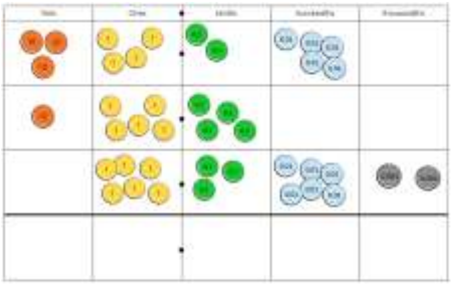
</td></tr>
</table>

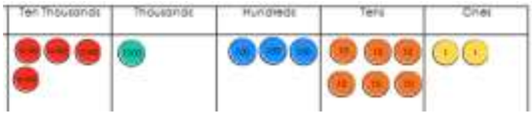
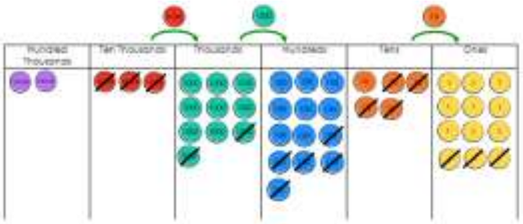
Strategies & Guidance	CPA
Partitioning one number and applying known facts to add. *Pupils can use this strategy mentally or with jottings as needed.* *Pupils should be aware of the range of choices available when deciding how to partition the number that is to be added.* *They should be encouraged to count on from the number of greater value as this will be more efficient. However, they should have an understanding of the commutative law of addition, that the parts can be added in any order.* *Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.*	**Partitioning into place value amounts (canonical partitioning):** 4650 + 7326 = 7326 + 4000 + 600 + 50  With place value counters, represent the larger number and then add each place value part of the other number. The image above shows the thousands being added. Represent pictorially with an empty numberline:  **Partitioning in different ways (non-canonical partitioning):** Extend the ‘Make ten’ strategy (see guidance in Y1 or Y2) to count on to a multiple of 10. 6785 + 2325 = 6785 + 15 + 200 + 2110  The strategy can be used with decimal numbers, Make one: 14.7 + 3.6 = 14.7 + 0.3 + 3.3 = 15 + 3.3 

Strategies & Guidance	**CPA**
Subtraction by partitioning and applying known facts. *Pupils can use this strategy mentally or with jottings as needed.* *Pupils should be aware of the range of choices available when deciding how to partition the number that is to be subtracted.* *Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.*	**Partitioning into place value amounts (canonical partitioning):** 75 221 – 14 300 = 75 221 – 10 000 – 4000 – 300  Represent pictorially with a number line, starting on the right and having the arrows jump to the left:  Develop understanding that the parts can be subtracted in any order and the result will be the same:  **Partitioning in different ways (non-canonical partitioning):** Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count back to a multiple of 10. 

Strategies & Guidance	CPA
Calculate difference by “counting back” *It is interesting to note that finding the difference is reversible. For example, the difference between 5 and 2 is the same as the difference between 2 and 5. This is not the case for other subtraction concepts.*	75 221 – 14 300 Place the numbers either end of a numberline and work out the difference between them. Select efficient jumps.  Finding the difference is efficient when the numbers are close to each other: 9012 – 8976 
Calculate difference by “counting on” *Addition strategies can be used to find difference.*	75 221 – 14 300  Finding the difference is efficient when the numbers are close to each other 9012 – 8976 

Strategies & Guidance	CPA
Round and adjust **Addition and subtraction using compensation** *Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding.* *It is very easy to be confused about how to adjust and so visual representations and logical reasoning are essential to success with this strategy.*	**Addition**  54 128 + 9987 = 54 128 + 10 000 – 13 = 64128 - 13 Pupils should realise that they can adjust first:  54128 + 9987 = 54128 – 13 + 10 000 = 54 115 + 10 000 **Subtraction**  78 051 – 9992 = 78 051 – 10 000 + 8 = 68 051 + 8 Pupils should realise that they can adjust first:  78 051 – 4960 = 78 051 + 40 – 5000 = 78 692 - 5000
Near doubles *Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten as well as decimal numbers. These facts can be adjusted to calculate near doubles.*	160 + 170 = double 150 + 10 + 20 160 + 170 = double 160 + 10 or 160 + 170 = double 170 - 10 2.5 + 2.6 = double 2.5 + 0.1

Strategies & Guidance	CPA
Partition both numbers and combine the parts *Pupils should be secure with this method for numbers up to 10 000, using place value counters or Dienes to show conceptual understanding.* *If multiple regroupings are required, then pupils should consider using the column method.*	7230 + 5310 = 12 000 + 500 + 40  Pupils should be aware that the parts can be added in any order.
Written column methods for addition *In Year 5, pupils are expected to be able to use formal written methods to add whole numbers with more than four digits as well as working with numbers with up to three decimal places.* *Pupils should think about whether this is the most efficient method, considering if mental methods would be more effective.* *Continue to use concrete manipulatives alongside the formal method.* *When adding decimal numbers with a different number of decimal places, in order to avoid calculation errors, pupils should be encouraged to insert zeros so that there is a digit in every row. This is not necessary for calculation and these zeros are not place holders as the value of the other digits is not changed by it being placed.* *Exemplification of this method and the language to use are best understood through viewing the tutorial videos found here on the toolkit.*	For this method start with the digit of least value because if regrouping happens it will affect the digits of greater value. 3 4 6 2 3 + 5 5 4 1  Combine the counters in each column and regroup as needed: 3 4 6 2 3 + 5 5 4 1 1 1 4 0 1 6 4  Decimal numbers: 3 4 . 2 5 1 5 . 4 + 6 . 3 6 2 1 1 1 5 6 . 0 1 2 

Strategies & Guidance	CPA
Written column methods for subtraction *In Year 5, pupils are expected to be able to use formal written methods to subtract whole numbers with more than four digits as well as working with numbers with up to three decimal places.* *Pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping.* *In Year 3 and 4 they become more familiar with calculations that require 'regrouping to regroup'. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.* *Pupils should think about if this is the most efficient method, considering whether mental strategies (such as counting on, using known number facts, compensation etc.) may be likelier to produce an accurate solution.* *Exemplification of this method and the language to use are best understood through viewing the tutorial videos found here on the toolkit.*	$\begin{array}{rrrrrr} & 4 & 1 & 3 & 6 & 2 \\ - & 3 & 2 & 2 & 4 & 3 \\ \hline & & & & & \\ \hline \end{array}$  $\begin{array}{rrrrrr} & \overset{3}{\cancel{4}} & {}^{1}1 & 3 & \overset{5}{\cancel{6}} & {}^{1}2 \\ - & 3 & 2 & 2 & 4 & 3 \\ \hline & & 9 & 1 & 1 & 9 \\ \hline \end{array}$  The term regrouping should be the language used. You can use the terms 'exchange' with subtraction but it needs careful consideration. You can regroup 62 as 50 and 12 (5 tens and 12 ones) instead of 60 and 2 (6 tens and 12 ones). Or you can 'exchange' one of the tens for 10 ones resulting in 5 tens and 12 ones. If you have exchanged, then the number has been regrouped.

Progression in calculations

Year 5 + Year 6

National Curriculum objectives linked to multiplication and division

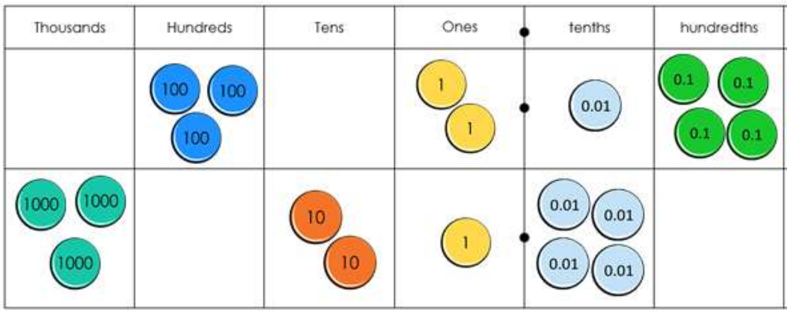
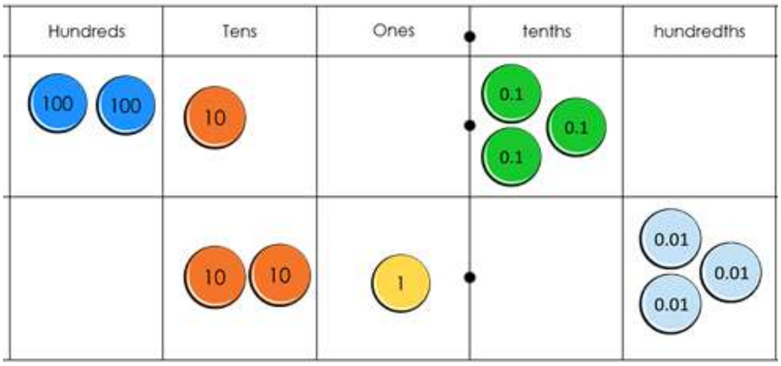
These objectives are explicitly covered through the strategies outlined in this document:

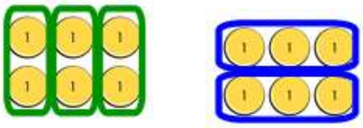
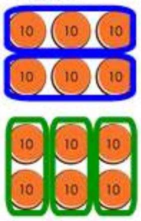
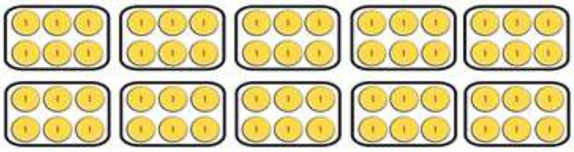
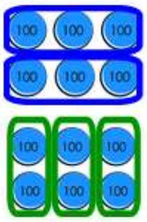
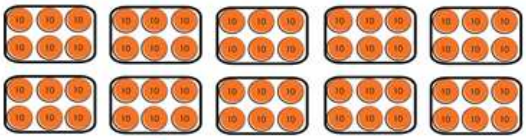
- multiply and divide whole numbers by 10, 100 and 1000
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts.

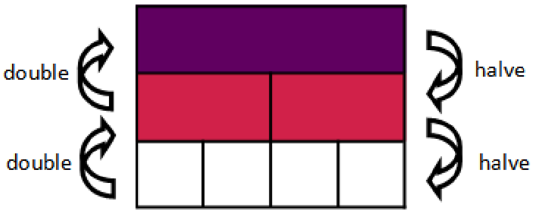
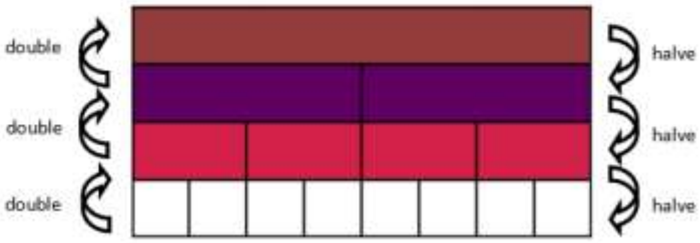
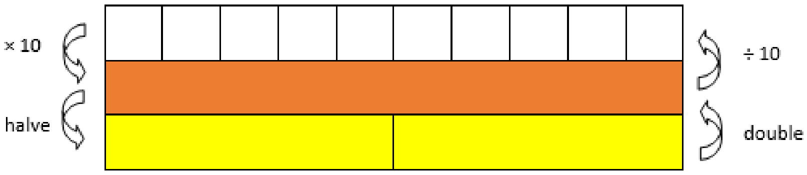
Y5 and Y6 Multiplication

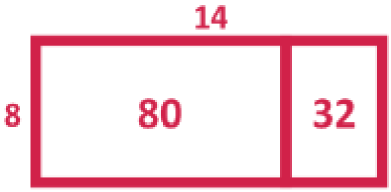
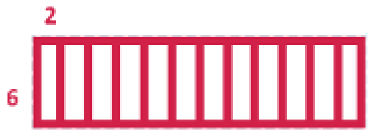
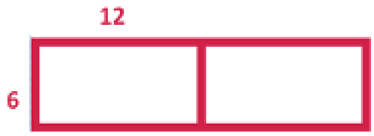
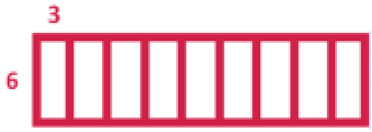
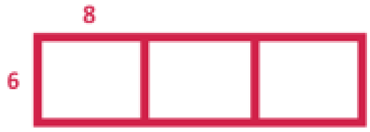
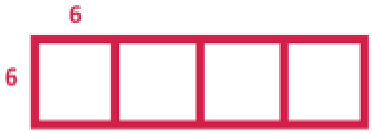
Strategies & Guidance	CPA
Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 *Avoid saying that you "add a zero" when multiplying by ten and instead use the language of place holder.* *Use place value counters and charts to visualise and then notice what happens to the digits.*	When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc. When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater. 102.14 x 10 = 1021.4  When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller. When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. 210 ÷ 10 = 21 210.3 ÷ 10 = 21.03 

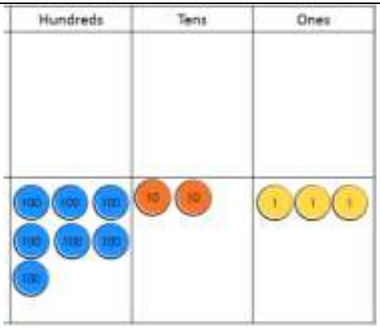
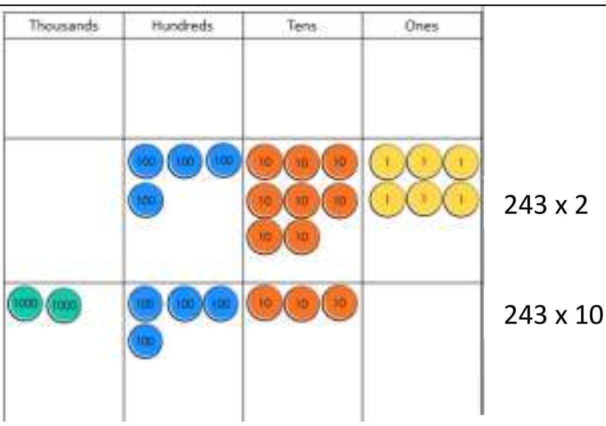
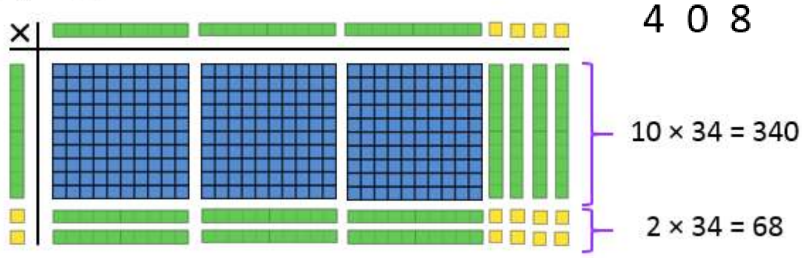
Strategies & Guidance	CPA
Using known facts and place value to derive multiplication facts *Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact.* *Knowledge of commutativity is further extended and applied to find a range of related facts.* *Pupils should work with decimals with up to two decimal places.* *These derived facts should be used to estimate and check answers to calculations.*	 2 x 3 = 6 3 x 2 = 6 2 x 30 = 60 30 x 2 = 60  3 x 20 = 60 20 x 3 = 60  2 x 3 x 10= 60 2 x 300 = 600 300 x 2 = 600  3 x 200 = 600 200 x 3 = 600 2 x 30 x 10 = 600 20 x 3 x 10 = 600   20 x 30 = 600 30 x 20 = 600

These are the multiplication facts pupils should be able to derive from a known fact

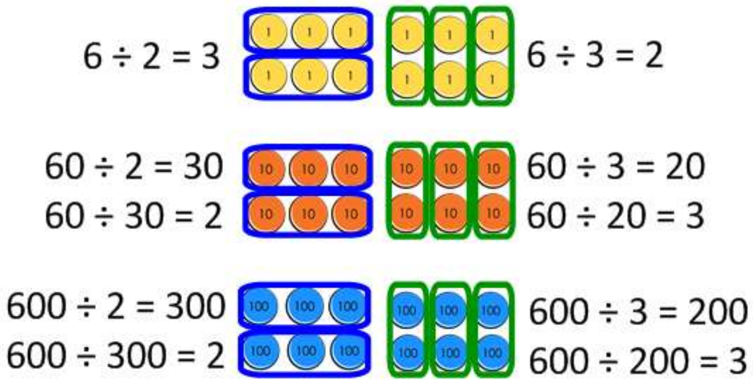
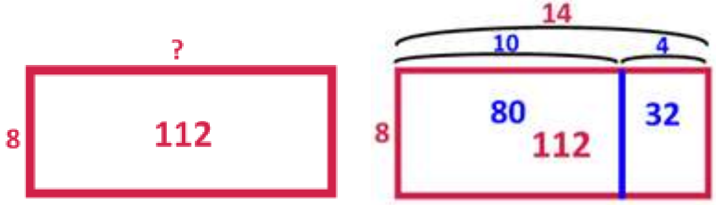
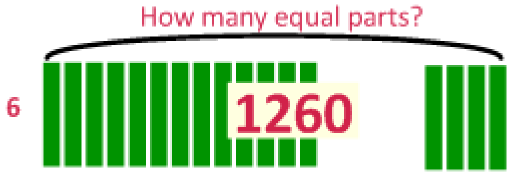
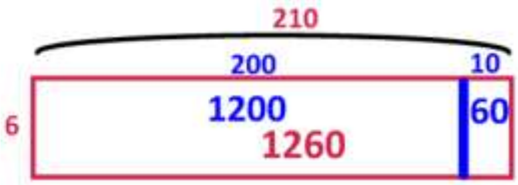
2 100 000		700 000 x 3	70 000 x 30	7000 x 300	700 x 3000	70 x 30 000	7 x 300 000
210 000		70 000 x 3	7000 x 30	700 x 300	70 x 3000	7 x 30 000	
21 000		7000 x 3	700 x 30	70 x 300	7 x 3000		
2100		700 x 3	70 x 30	7 x 300			
210		70 x 3	7 x 30				
21	=	**7 x 3**					
2.1		0.7 x 3	7 x 0.3				
0.21		0.07 x 3	0.7 x 0.3	7 x 0.03			
0.021		0.007 x 3	0.07 x 0.3	0.7 x 0.03	7 x 0.003		

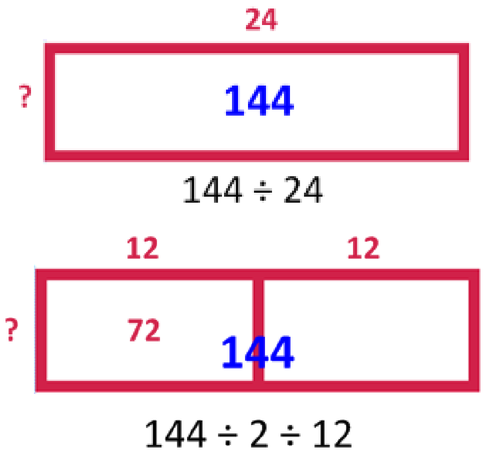
Strategies & Guidance	CPA
Doubling and halving *Pupils should experience doubling and halving larger and smaller numbers as they expand their understanding of the number system.* *Doubling and halving can then be used in larger calculations.*	 **Multiply by 4** by doubling and doubling again e.g. 16 × 4 = 32 × 2 = 64 **Divide by 4** by halving and halving again e.g. 104 ÷ 4 = 52 ÷ 2 = 26  **Multiply by 8** by doubling three times e.g. 12 × 8 = 24 × 4 = 48 × 2 = 96 **Divide by 8** by halving three times e.g. 104 ÷ 8 = 52 ÷ 4 = 26 ÷ 2 = 13  **Multiply by 5** by multiplying by 10 then halving, e.g. 18 × 5 = 180 ÷ 2 = 90. **Divide by 5** by dividing by 10 and doubling, e.g. 460 ÷ 5 = double 46 = 92

Strategies & Guidance	CPA
Multiply by partitioning one number and multiplying each part **Distributive law** **a x (b + c) = a x b + a x c** Build on pupils' understanding of arrays of counters to represent multiplication to see that area models can be a useful representation:	8 x 14 = 8 x 10 + 8 x 4  Cuisenaire rods to build arrays  Represent with area model  Jottings on a number line Bead string where each bead has a value of 8: 
Using knowledge of factors *In Year 5 pupils are expected to be able to identify factor pairs and this knowledge can be used to calculate.* *Pupils will be using the commutative and associative laws of multiplication.* **Commutative law** **a x b = b x a** **Associative law** **a x b x c = (a x b) x c** **= a x (b x c)** *They should explore and compare the different options and choose the most efficient order to complete calculations.*	Calculate 6 x 24 by using factor pairs of 24 Two and twelve are factors of 24: 6 x 2 x 12  6 x 12 x 2  Three and eight are factors of 24: 6 x 3 x 8  6 x 8 x 3  Four and six are factors of 24: 6 x 4 x 6  6 x 6 x 4 

Strategies & Guidance	CPA
Formal written method of short multiplication *Conceptual understanding is supported by the use of place value counters, both during teacher demonstrations and during their own practice.* *Exemplification of this method and the language to use are best understood through viewing the tutorial videos found here on the toolkit.*	241 x 3 723 1 
Multiplying by a 2-digit number **Formal written method of long multiplication** *In Year 6 pupils are extended from multiplication by a 1-digit number to multiplication by a 2-digit number.* *Extend the place value chart model used in Year 4, using an additional row on the place value chart.* *Extend understanding of the distrubitive law to develop conceptual understanding of the two rows of the formal written method.* *Dienes blocks can be used to construct area models to represent this.*	243 x 12 486 2430  3 4 × 1 2 6 8 3 4 0 4 0 8 12 × 34 

Y5 and Y6 Division

Strategies & Guidance	CPA
Deriving facts from known facts *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.*	
Using knowledge of multiples to divide *Using an area model to partition the whole into multiples of the divisor (the number you are dividing by).*	 112 ÷ 8 = 80 ÷ 8 + 32 ÷ 8  1260 ÷ 6 = 1200 ÷ 6 + 60 ÷ 6  

Strategies & Guidance	**CPA**
Using knowledge of factors to divide *Pupils explore this strategy when using repeated halving.* *2 x 2 = 4 and so if you divide by 4 the same result can be achieved by dividing by two and then by two again.*	 I know 2 and 12 are a factor pair of 24 and so I can divide by 2 and then by 12.

<table>
<tr><th>Strategies & Guidance</th><th>CPA</th></tr>
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Short division

Dividing a 4-digit numbers by 1-digit numbers

The thought process of the traditional algorithm is as follows:

How many 4s in 8? 2
How many 4s in 5? 1 with 1 remaining so regroup.
How many 4s in 12? 3
How many 4s in 8? 2

Warning: If you simply apply place value knowledge to each step, the thinking goes wrong if you have to regroup.

How many 4s in 8000? 2000
How many 4s in 500?
100 with 1 remaining (illogical)
The answer would be 125.

Sharing the dividend builds conceptual understanding however doesn't scaffold the "thinking" of the algorithm.

Using place value counters and finding groups of the divisor for each power of ten will build conceptual understanding of the short division algorithm.

Area models are also useful representations, as seen with other strategies and exemplified for long division.

Exemplification of this method and the language to use are best understood through viewing the tutorial videos found here on the toolkit.

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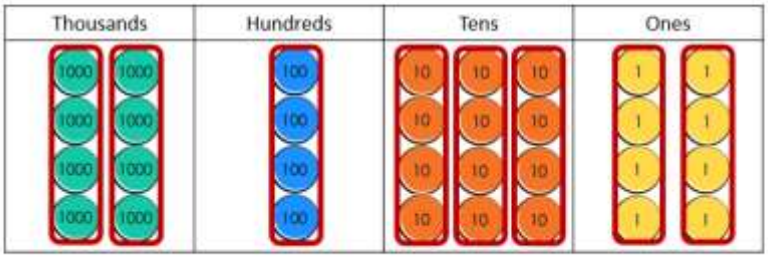
8528 ÷ 4

$$\begin{array}{r|l} & 2\;1\;3\;2 \\ \hline 4 & 8\;5^{1}2\;8 \end{array}$$

Sharing

8 thousands shared into 4 equal groups
5 hundreds shared into 4 equal groups
Regroup 1 hundred for 10 tens
12 tens shared into 4 equal groups
8 ones shared into 4 equal groups.

Grouping



How many groups of 4 thousands in 8 thousands?
How many groups of 4 hundreds in 5 hundreds?
Regroup 1 hundred for 10 tens.
How many groups of 4 tens in 12 tens?
How many groups of 4 ones in 8 ones?

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Strategies & Guidance	**CPA**
Long division **Dividing a 4-digit number by a 2-digit number** *Follow the language structures of the short division strategy. Instead of recording the regrouped amounts as small digits the numbers are written out below. This can be easier to work with when dividing by larger numbers.* *If dividing by a number outside of their known facts, pupils should start by recording some multiples of that number to scaffold.*	$\begin{array}{r} 34 \\ 12\overline{)408} \\ \underline{36}\downarrow \\ 48 \\ \underline{48} \\ 0 \end{array}$ 408 ÷ 12